Reading Questions
Math 434, Abstract Algebra II
Spring 2013

Chapter 16, Rings

1. What is the fundamental difference between groups and rings?
2. Give two characterizations of an integral domain.
3. Provide two examples of fields, one infinite, one finite.
4. Who was Emmy Noether?
5. Speculate on a computer program that might use the Chinese Remainder Theorem to speed up computations with large integers.

Chapter 17, Polynomials

1. Suppose $p(x)$ is a polynomial of degree n with coefficients from any field. How many roots can $p(x)$ have? How does this generalize your high school algebra experience?
2. What is the definition of an irreducible polynomial?
3. Find the remainder upon division of $8\,x^5 - 18\,x^4 + 20\,x^3 - 25\,x^2 + 20$ by $4\,x^2 - x - 2$.
4. A single theorem in this chapter connects many of the ideas of this chapter to many of the ideas of the previous chapter. State a paraphrased version of this theorem.
5. Early in this chapter, Judson says, "We can prove many results for polynomial rings that are similar to the theorems we proved for the integers." Write a short essay (or a very long paragraph) justifying this assertion.

Chapter 18, Integral Domains

1. Integral domains are an abstraction of which two fundamental rings that we have already studied?
2. What are the various types of integral domains defined in this section?
3. The field of fractions of a ring abstracts what idea from basic mathematics?
4. Which theorem in this chapter generalizes Theorem 13 from the previous chapter?
5. Describe an example which is a UFD, but not a PID.

Chapter 19, Lattices and Boolean Algebras

1. Describe succinctly what a poset is. Do not just list the defining properties, but give a description that another student of algebra who has never seen a poset might understand. For example, part of your answer might include what type of common algebraic topics a poset generalizes, and your answer should be short on symbols.
2. How does a lattice differ from a poset? Answer this in the spirit of the previous question.
3. How does a Boolean algebra differ from a lattice? Again, answer this in the spirit of the previous two questions.
4. Give two (perhaps related) reasons why any discussion of finite Boolean algebras might center on the example of the power set of a finite set.
5. Describe a major innovation of the middle 20th century made possible by Boolean algebra.

Chapter 20, Vector Spaces

1. Why do the axioms of a vector space appear to only have four condtitions, rather than the ten you may have seen the first time you saw an axiomatic definition?
2. $V = \mathbb{Q}(\sqrt{11}) = \left\{a + b\sqrt{11} \mid a, b \in \mathbb{Q}\right\}$ is a vector space. Carefully define the operations on this set that will make this possible. Describe the subspace spanned by $S = \{\mathbf{u}\}$, where $\mathbf{u} = 3 + \frac{2}{7}\sqrt{11} \in V$.
3. Write a long paragraph, or a short essay, on the importance of linear independence in linear algebra.
4. Write a long paragraph, or a short essay, on the importance of spanning sets in linear algebra.
5. "Linear algebra is all about linear combinations." Explain why you might say this.

Chapter 21, Fields

1. What does it mean for an extension field E of a field F to be a simple extension of F?
2. What is the minimal polynomial of an element $\alpha \in E$, where E is an extension of F, and α is algebraic over F?
3. Describe how linear algebra enters into this chapter. What critical result relies on a proof that is almost entirely linear algebra?
4. When is a field algebraically closed?
5. What is a splitting field of a polynomial $p(x) \in F[x]$?

Chapter 22, Finite Fields

1. When is a field extension seperable?
2. What are the possible orders for subfields of a finite field?
3. What is the structure of the non-zero elements of a finite field?
4. Provide a characterization of finite fields using the concept of a splitting field.
5. Why is a theorem in this chapter titled “The Freshman’s Dream?”

Chapter 23, Galois Theory

1. What is the Galois group of a field extension?
2. When are two elements of a field extension conjugate?
3. Summarize the nature and importance of the Fundamental Theorem of Galois Theory. Capture the essence of the result without getting bogged down in too many details.
4. Why are “solvable” groups so named? Paraphrasing the relevant theorem would be a good answer.
5. Argue the following statement, both pro and con. Which side wins the debate?

   Everything we have done all year long has been in preparation for this chapter.