Math 290 Name: Key Dr. Beezer
Exam 1 Spring 2014
Chapter SLE

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to row-reduce matrices, except in the question that asks you to row-reduce without Sage. No other use of Sage may be used as justification for your answers.

1. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$\begin{aligned} -2x_1 + 3x_2 - 2x_3 + x_4 &= 1 \\ -x_1 + x_2 - x_3 - x_4 &= 0 \\ 3x_1 - 3x_2 + 4x_3 + 6x_4 &= 1 \\ 3x_2 - 2x_3 + 4x_4 &= 5 \end{aligned}$$

Augmented matrix representing the system

$$\left[\begin{array}{cccc|c} -2 & 3 & -2 & 1 & 1 \\ -1 & 1 & -1 & -1 & 0 \\ 3 & -3 & 4 & 6 & 1 \\ 0 & 3 & -2 & 4 & 5 \end{array}\right]$$

rref →

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & -11 \\ 0 & 0 & 1 & 0 & -11 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

Consistent, $n = 4 = r \Rightarrow$ unique solution

$$S = \left\{ \begin{bmatrix} -4 \\ -11 \\ -11 \\ 4 \end{bmatrix} \right\}$$

2. Solve the following system of linear equations and express the solutions as a set of column vectors. (20 points)

$$\begin{aligned} x_1 + 2x_2 + 7x_3 + 6x_4 - 3x_5 &= 12 \\ x_1 - 3x_3 - x_4 &= -1 \\ -x_2 - 5x_3 - 4x_4 + 2x_5 &= -8 \end{aligned}$$

Augmented matrix representing the system

$$\left[\begin{array}{ccccc|c} 1 & 2 & 7 & 6 & -3 & 12 \\ 1 & 0 & -3 & -1 & 0 & -1 \\ 0 & -1 & -5 & -4 & 2 & -8 \end{array}\right]$$

rref →

$$\left[\begin{array}{ccccc|c} 1 & 0 & -3 & 0 & -1 & 2 \\ 0 & 1 & 5 & 0 & 2 & -4 \\ 0 & 0 & 0 & 1 & -1 & 3 \end{array}\right]$$

$x_1 = 2 + 3x_3 + x_5$
$x_2 = -4 - 5x_3 - 2x_5$
$x_4 = 3 + x_5$

$D = \{1, 2, 4\}$ $F = \{3, 5, 6\}$ ← so system is consistent

$$S = \left\{ \begin{bmatrix} 2 + 3x_3 + x_5 \\ -4 - 5x_3 - 2x_5 \\ x_3 \\ 3 + x_5 \\ x_5 \end{bmatrix} \middle| x_3, x_5 \in \mathbb{C} \right\}$$

3. Without using Sage, find a matrix B in reduced row-echelon form which is row-equivalent to A. It is especially important to show all of your work, so it is clear you have not used Sage. (20 points)

$$A = \begin{bmatrix} 1 & -2 & -4 & -6 \\ 2 & -3 & -5 & -11 \\ 1 & 2 & 8 & -2 \end{bmatrix} \xrightarrow[-2R_1+R_2]{-R_1+R_3} \begin{bmatrix} 1 & -2 & -4 & -6 \\ 0 & 1 & 3 & 1 \\ 0 & 4 & 12 & 4 \end{bmatrix} \xrightarrow[-4R_2+R_3]{2R_2+R_1} \begin{bmatrix} 1 & 0 & 2 & -4 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

B

4. Determine if the matrix below is nonsingular or singular. Explain your reasoning carefully and thoroughly. (15 points)

$$\begin{bmatrix} 1 & 1 & 0 & 2 & 2 & -4 & 5 \\ -1 & 0 & 1 & 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 1 & 2 & -7 & 6 \\ 1 & 1 & -1 & 2 & 2 & -1 & 1 \\ -1 & -2 & 0 & -5 & -3 & 4 & -7 \\ 2 & 1 & 0 & 1 & 3 & -7 & 6 \\ 1 & 1 & -2 & 0 & 1 & 4 & -3 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

By Theorem NMRRI, the matrix is nonsingular.

5. Compute the null space of the matrix D. (15 points)

$$D = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -2 & -3 & 3 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

If D were the coefficient matrix of the homogeneous system $LS(D, \underset{\sim}{0})$ an equivalent system is

$$x_1 - 3x_3 - 2x_4 = 0 \longrightarrow x_1 = 3x_3 + 2x_4$$

$$x_2 + x_3 = 0 \longrightarrow x_2 = -x_3$$

Null space = solution set to $LS(D, \underset{\sim}{0})$

$$= \left\{ \begin{bmatrix} 3x_3 + 2x_4 \\ -x_3 \\ x_3 \\ x_4 \end{bmatrix} \middle| \; x_3, x_4 \in \mathbb{C} \right\}$$

6. Suppose that A is the augmented matrix of a system of equations with n variables. Suppose that B is a matrix that is row-equivalent to A and is in reduced row-echelon form, with $r = n + 1$. Give a careful, well-written, proof that the system of equations is inconsistent. (15 points)

(1) r = # pivot columns

(2) $n+1$ = total number of columns in augmented matrix A & so also in B.

(3) $r = n+1 \Rightarrow$ every column is a pivot column

(4) So the "last" column is a pivot column

(5) By RCLS, the system is inconsistent.