Computer Vision for Linear Algebra

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What is Computer Vision?

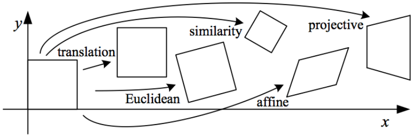
- Artificial intelligence
- Having a computer describe what is sees in the world around it the way a human or animal can
- An inverse problem recovering unknowns from insufficient data to fully describe a “solution”

Homogenous coordinates

- Adding an extra dimension to the Euclidean system
- Account for the concept of infinity

Transformations

- Linear transformations used to manipulate the image
- Represented through matrices which act on image vectors through multiplication
- Camera projection matrix
- Types of transformations:



- Real world data is error prone
- Total Least Squares
- Robust Least Squares
- Non-Linear Least Squares

- Rotation, scaling, rotation
- Principal Component Analysis
- Generalized inverse

- Most situations can be approximated well linearly
- Estimation of lost data

Matrix Decompositions

- Inverting the transformation from 3D to 2D
- Speedy Calculations
- Accurate Computations
- Projections
- Positive Definite Matricies

Efficiency

- Space efficiency
- Time efficiency

Modern Uses of Computer Vision

Structure from Motion

- Process of using a series of 2D images to reconstruct a 3D image
- Depth has been lost in a single 2D image
- Transformations that cannot be inverted have occurred in the process of shifting from 3D to 2D
- Each camera has its own coordinate system so the stationary coordinate system of the object must be used

Method

- Clearly identifiable points are identified on the images
- The generalized inverse of the camera's projection matrix is calculated for a single image
- The attempt at an inverted image is transformed into the system of the second image
- The location of identified points in the second image are compared to those of the transformed image
- Additional transformations are estimated to better align the key points
- The process is repeated until key points are reasonably well aligned

Example


crisp·clean
refreshing
100% natural
flavors
7UP
Naturally Flavored Soda
7UP
7UP

Convolution

- Form of linear filtering
- Utilizes a convolution kernel to perform an action on a larger image matrix
- Examples: Blurring, Sharpening, Smoothing, Edge-Identifying, and more

Method

- Find the appropriate $n \times n$ convolution kernel
- For every pixel in the image matrix form an $n \times n$ sub-matrix with the adjacent pixels.
- Define the following convolution action on a segment of the image matrix, I, by the convolution kernel K as:

$$b = \sum_{i=0}^{n} \sum_{j=0}^{n} I_{i,j} K_{i,j}$$

- Replace each pixel in the image matrix with the convolution action of the pixel's sub matrix.

Example

Original	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
Edge-Detect	$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	
	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	
	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	

Sharpen	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
Blur*	$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$	
	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	

Eigenfaces

- Used for face identification and recognition
- Eigenfaces are the eigenvectors of the covariance matrix for a set of face images.
- Each face image is a linear combination of the eigenfaces
- Only the "most important" eigenfaces are stored

Method

- Take a set of $n \times m$ face picture matrices and treat them as a set of nm dimensional vectors.

$$\begin{bmatrix} i_{1,1} & \dots & i_{n,1} \\ \dots & \dots & \dots \\ i_{1,m} & \dots & i_{n,m} \end{bmatrix} \rightarrow \begin{bmatrix} i_{1,1} \\ \dots \\ i_{n,1} \\ i_{1,2} \\ \dots \\ i_{n,m} \end{bmatrix} = \mathbf{i}$$

- Find the average face: $\mathbf{a} = \frac{1}{m} \sum_{j=1}^{m} \mathbf{i_j}$
- Form the image matrix with columns of the difference between each face vector and the average face. $\mathbf{i_j} - \mathbf{a}$
- The right singular vectors of this matrix are the eigenfaces

Face Recognition

- Treat the face picture matrix as a face vector, $\mathbf{f}$.
- Find the face vector's eigenface components: $w_i = \mathbf{e_i^*}(\mathbf{f} - \mathbf{a})$
- Form these eigenface components into a component vector, $\mathbf{w}$
- Calculate the Euclidian distance between the unknown face's component vector and each known face's component vectors as the square of their inner product.
- The face which minimizes the Euclidian distance is the one to which the unknown face belongs

Example

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