Show all of your work and explain your answers fully. There is a total of 100 possible points.

For computational problems, place your answer in the provided boxes. Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices, except in the question that asks you to row-reduce without Sage. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$x_1-2x_2+5x_3=-2$$
 Form the augmental matrix  $x_2-2x_3+x_4=1$  and vow-veduce (w/Sage)

Since Column 5 is

[1 -2 5 0 | -2] PREF (00 | 2 | 0) a pivot Column, by

[0 1 -2 1 | 1] 5 [00-2 | 0] Theorem RUS the system is

[2 3 -4 7 | 5] [0 0 0 0 0] incarsistent.

0 = 24

2. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$x_1 - 2x_2 - 4x_3 + 3x_4 + 4x_5 = 1$$

$$x_2 + 2x_3 - 2x_4 - x_5 = 1$$

$$-x_1 + 3x_2 + 6x_3 - 5x_4 - 5x_5 = 0$$

$$2x_1 - 3x_2 - 6x_3 + 4x_4 + 7x_5 = 3$$

Augmental materix, row-reduce of Sage 000-1213

Consistent by Theaen RUS. r=2 pivot columns. X3, X4, X5 fre

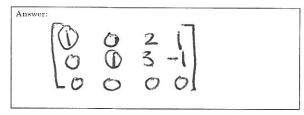
$$X_1 = 3 + x_4 - 2x_5$$
  
 $X_2 = 1 - 2x_3 + 2x_4 + x_5$ 

Answer: 
$$S = \begin{cases} 3 + x_4 - 2x_5 \\ 1 - 2x_3 + 2x_4 + x_5 \\ x_3 \\ x_4 \\ x_5 \end{cases} \begin{cases} x_3, x_4 \\ x_5 \in C \end{cases}$$

3. Without using Sage, find a matrix B in reduced row-echelon form which is row-equivalent to A. It is especially important to show all of your work, so it is clear you have not used Sage. (20 points)

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & 7 & 1 \\ 3 & 2 & 12 & 1 \end{bmatrix} \quad \begin{array}{c} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array} \quad \begin{array}{c} (1) & 0 & 2 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 2 & 6 & -2 \end{array}$$

$$-2R_2+R_3$$
  $\begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ 



4. Determine if the matrix below is nonsingular or singular. Explain your reasoning carefully and thoroughly. (15 points)

$$\begin{bmatrix} 3 & 4 & -1 & 6 & 8 \\ 0 & 1 & -1 & -1 & -5 \\ 2 & 5 & -2 & 4 & -1 \\ 4 & 4 & -1 & 6 & 7 \\ 2 & 3 & -1 & 4 & 5 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} = \text{Ts} \quad \text{the mestax is} \quad \text{nonsingular.}$$

- 5. Say as much as possible about the solution set of each system, along with justifications for your answers. (15 points)
  - (a) Homogeneous, 5 variables and 8 equations.

(b) Coefficient matrix is nonsingular.

(c) 8 variables, 5 equations.

6. Suppose that the coefficient matrix of a homogeneous system has two columns that are identical. Prove that the system has infinitely many solutions. (15 points)

Suppose columns K & l are identical.

1) Homogeneous => one solution, at least.

Show that 
$$X_1 = \lambda_2 = \cdots = X_{k-1} = 0$$
,  $X_k = 1$ ,  $X_{kn} = \cdots = X_{k-1} = 0$ ,  $X_k = -1$ ,  $X_{kn} = \cdots = X_{k-1} = 0$ 

is a second solution, hence infinitely many.

OR

2) Row-reduce the mosture of columns Kal will remain equal. They cannot both be pirot columns, so then is at least one non-pirot

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Many Si			consistent		