Math 280: Multivariate Calculus

I. Introduction

A. Catalog Description

Continuing through the calculus sequence that starts with MATH 180 and 181, is an introduction to the study of functions that have several variable inputs and/or outputs. The central ideas involving these functions are explored from the symbolic, the graphic, and the numeric points of view. Visualization and approximation, as well as local linearity continue as key themes in the course. Topics include vectors and the basic analytic geometry of three-space; the differential calculus of scalar-input, vector-output functions; the geometry of curves and surfaces; and the differential and integral calculus of vector-input, scalar-output functions. *Prerequisite: MATH 181 or its equivalent.* Satisfies the Mathematical Approaches core requirement.

B. Objectives

The primary objective for students in this course is to appreciate the power and beauty of the calculus. In this multi-variable setting, students will begin to appreciate the central role of linearity. They will also see how approximation by polynomial functions is fundamental for understanding optimization problems. In the study of vector-input, vector-output functions, students will see how abstract concepts lead to a deeper understanding of important mathematical ideas. In particular, they will begin to experience the organic and highly interconnected nature of mathematics by using calculus to analyze and solve problems from the sciences and business related fields.

This course satisfies the Mathematical Approaches category of the university's core curriculum by developing an appreciation of the power of mathematics and formal methods to provide a way of understanding a problem unambiguously, describing its relation to other problems, and specifying clearly an approach to its solution. A student in this course will develop a variety of mathematical skills, an understanding of formal reasoning, and a facility with applications. Specifically, this course will expose students to formal logic to the extent that it is required to understand mathematical proof.

C. Learning Outcomes

- Be comfortabe with two, three, and higher-dimensional spaces as well as vectors and their operations (e.g. the dot and cross products);
- To differentiate and integrate vector-valued functions in three-dimensional space;
- To define and compute partial derivatives and use them for optimization problems involving functions of several variables;
- Evaluate multiple integrals, include using polar, cylindrical, and spherical coordinates;
- Recognize vector-valued functions to analyze curves and their motion (e.g. velocity and acceleration vectors);
- Identify vector fields, evaluate line integrals, and understand the connections between conservative vector fields and path-independence;

- Understand multivariate calculus' central theorems (e.g. Fundamental Theorem for Line Integrals and Green's theorem); and
- Communicate mathematical concepts in well-written, logical, and concise ways.

D. Prerequitites - Math 181 - Calculus and Analytic Geometry II, or its equivalent

II. Course Syllabus

A. Vectors and Three-Dimensional Geometry

1. Three-Dimensional Cartesian Coordinates
2. Vectors
3. The Dot Product
4. The Cross Product
5. Equations of Lines and Planes
6. A Survey of Quadratic Spaces

B. Coordinate Systems in Two- and Three-Dimensions

1. Polar Coordinates
2. Area and Arc Length in Polar Coordinates
3. Cylindrical Coordinates
4. Spherical Coordinates

C. Vector-Valued Functions and Their Calculus

1. Vector-Valued Functions
2. Derivatives and Integrals of Vector-Valued Functions
3. Speed and Arc Length
4. Curvature *Optional*
5. Motion in Three-Dimensions *Optional*

D. Partial Derivatives

1. Functions of Several Variables
2. Limits and Continuity in Several Variables
3. Partial Derivatives
4. Tangent Planes
5. The Chain Rule
6. Directional and Gradient Derivatives
7. Maximum and Minimum Values
8. Lagrange Multipliers *Optional*

E. Multiple Integrals

1. Double Integrals over Rectangles
2. Iterated Integrals
3. Double Integrals over General Regions
4. Triple Integrals
5. Integrals over Polar, Cylindrical, and Spherical Coordinates

6. Change of Variables in Multiple Integrals *Optional*

F. **Vector Fields and Line Integrals**
   1. Vector Fields
   2. Line Integrals
   3. Conservative Vector Fields
   4. The Fundamental Theorem for Line Integrals
   5. Green's Theorem

G. **Surface Integrals**
   1. Parametrized Surfaces *Optional*
   2. Surface Integrals *Optional*
   3. Stokes' Theorem
   4. The Divergence Theorem

III. **Bibliography**

- Jon Rogawski & Colin Adams. *Calculus: Early Transcendentals*.
- James Stewart. *Calculus: Early Transcendentals*.
- Deborah Hughes-Hallett, Andrew Gleason, William McCallum et al. *Calculus: Single and Multivariable*.
- Susan Colley. *Vector Calculus*.
- Jerrold E. Marsden & Anthony Tromba. *Vector Calculus*.