Writing Exercises

Math 290 Fall 2019

Dr. Beezer

Specific Instructions:

- Use CoCalc to make a PDF (links on course page). You may wish to make local copies to preserve your work.
- Include your name on each page. Do not print anything on the back side of any of page. **Staple** your work together, with this cover sheet on top, and each problem stapled individually.
- Examples are not proofs, and may not be used as justifications for general statements.
- Your audience is your peers in this course, not me. You may quote acronyms without providing any kind of reference. This means repeating whole theorems or definitions verbatim from the text is not necessary and is counter-productive.
- Write with complete sentences. You are **required** to use LaTeX as necessary. For example, **every** single symbol should be treated as math. So a matrix A would be surrounded by dollar signs. All vectors need to be bold ($\mathbf{v}$) or need to have arrows ($\vec{v}$). Alignment of equations will be enforced.
- Only use theorems and definitions from the relevant chapter, or earlier. This applies to retries as well.

General Reminders and Hints:

1. Don't ever use contractions in formal writing. Everything your English teachers taught you applies here.
2. Do not use symbols in place of words, nor words in place of symbols.
   - Bad: We see that x is $\leq$ to 3.
   - Better: We see that $x \leq 3$.
   - Bad: The number from above is much greater than the number we just saw.
   - Better: Thus x is much greater than y.
3. Use LaTeX's math-mode for every symbol, even if it is just a single letter, so use `\(x$\)` for the variable x.
4. Vectors must be typeset differently: `\(\vec{v}\)` or `\(\mathbf{v}\)`.
5. "A 3×4 matrix" can be accomplished with `\(3\times 4\)`.
6. `\usepackage{amsmath}` and `\usepackage{amssymb}` are necessary in the preamble to enable many extra mathematical commands.

7. For a matrix try

```
$A=\begin{bmatrix}
a & b & c & d\\
e & f & g & h\\
i & j & k & \ell
\end{bmatrix}$
```

to get $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & \ell \end{bmatrix}$

8. For a proof with reasons try

```
\begin{align*}
x^2 - y^2 &= x^2 -xy + xy -y^2  &&\text{Adding zero}\\
          &= x(x-y) + y(x-y)    &&\text{Distributivity, twice}\\
          &= (x+y)(x-y)         &&\text{Distribute } x-y
\end{align*}
```

to get

$$\begin{aligned}
x^2 - y^2 &= x^2 - xy + xy - y^2 && \text{Adding zero}\\
&= x(x-y) + y(x-y) && \text{Distributivity, twice}\\
&= (x+y)(x-y) && \text{Distribute } x-y
\end{aligned}$$

9. When your LaTeX gets super-complicated and has errors in it, "comment-out" every line of the offending region by using a percent-sign (%) at the start of the line, until the errors go away. This will make the line a "comment" that LaTeX will not process. Then slowly add back in new lines until the error surfaces and you can pinpoint just where.

Staple X me X	Math 290 Proof SLE-1	Name:	Dr. R. Beezer Fall 2019

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SLE-1 (Section HSE)

Suppose that the coefficient matrix of a homogeneous system of equations has a column of zeros. Prove that the system has infinitely many solutions.
Hint: What are the possibilities for the number of solutions to a linear system of equations? Can you definitively rule out any of these?

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SLE-2 (Section NM)

Suppose that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix where $ad - bc \neq 0$. Prove that A is nonsingular.

Hints: An example will not constitute a proof. One approach to a proof is to consider two general cases, $a = 0$ and $a \neq 0$. No matter what approach you choose, make sure you are never ever dividing by a variable quantity that could be zero.

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V-1 (Section VO)

Prove Property AAC of Theorem VSPCV. That is:

If $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w} \in \mathbb{C}^m$, then $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$.

Write your own proof in the style of proofs of Property DSAC (Theorem VSPCV) and Property CC (Solution VO.T13).
Hints: Think carefully about the two types of equality you will likely use in a proof. A successful proof will likely conclude with an appeal to Definition CVE.

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V-2 (Section LI)

Prove that the set of standard unit vectors (Definition SUV) is linearly independent.
Hints: There are several ways to do this, some easier than others. If you choose an approach that claims two matrices are equal, then you must establish that equality very carefully (how do we do that?). In other approaches, be very careful that the hypotheses of any theorems you use are satisfied.

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Staple X me X	Math 290 Proof M-1	Name:	Dr. R. Beezer Fall 2019

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M-1 (Section MM)

Suppose that A is an $m \times n$ matrix with a row where every entry is zero. Suppose that B is an $n \times p$ matrix. Prove that AB has a row where every entry is zero.
Hints: Theorem EMP should be useful, and you want to be explicit about which row of A has the zeros and which row of AB has the zeros. Which row is all zeros? Does it have a name?
It is highly unlikely you can prove this without relying heavily on symbols and equations.

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M-2 (Section MM)

Use Theorem EMP to prove part (2) of Theorem MMIM: If A is an $m \times n$ matrix and I_m is the identity matrix of size m, then $I_m A = A$.
Hint: Carefully studying the proof of part (1) of Theorem MMIM would be a good way to start.

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VS-1 (Section S)

Give an example of using Theorem TSS by proving that $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \middle| \, 5x_1 + 3x_2 - 8x_3 + 2x_4 = 0 \right\}$ is a subspace of $\mathbb{C}^4$.

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Staple X me X	Math 290 Proof VS-2	Name:	Dr. R. Beezer Fall 2019

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VS-2 (Section PD)

Carefully read Exercise PD.T60 and its solution (Solution PD.T60). Prove the "more general" result given in the solution, using the book's solution as a model.

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Staple X me X	Math 290 Proof D&E-1	Name:	Dr. R. Beezer Fall 2019

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D&E-1 (Section DM)

Prove that the inverse of an elementary matrix is a single elementary matrix.
Hints: Seek inspiration from Exercise RREF.T10
If you are tempted to "cancel" a matrix, you may want to think again.
This theorem does not say an elementary matrix is invertible — it says the inverse of an elementary matrix has the form of an elementary matrix — just one. How would you convince somebody of that?
There are three different types of elementary matrices, yes?
A well-written argument will likely consider both the *function* and the *form* of an elementary matrix.

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D&E-2 (Section EE)

Suppose that A is a matrix that is equal to its inverse, $A = A^{-1}$. Prove that the only possible eigenvalues of A are $\lambda = 1$ and $\lambda = -1$. Give an example of matrix that is equal to its inverse and actually has both of these possible values as eigenvalues.
Hints: Theorem EIM says that if λ is an eigenvalue of A, then λ^{-1} is an eigenvalue of A^{-1}. The hypothesis that $A = A^{-1}$ does not allow you to *immediately* conclude that $\lambda = \lambda^{-1}$. Also, Theorem EIM makes no statements about eigenvectors. However, you might study the proof of Theorem EIM for useful techniques. If you are tempted to "cancel" a vector, you need a theorem to back it up.

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LT-1 (Section LT)

Prove that the function

$$T\colon \mathbb{C}^3 \to \mathbb{C}^2, \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 + 5x_3 \\ 3x_1 + 8x_3 \end{bmatrix}$$

is a linear transformation.

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LT-2 (Section IVLT)

Suppose $T\colon U \to V$ is a surjective linear transformation and $\dim(U) = \dim(V)$. Prove that T is an invertible linear transformation. Write your proof in a style that mimics proofs in the textbook.

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Staple X me X	Math 290 Proof R-1	Name:	Dr. R. Beezer Fall 2019

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R-1 (Section MR)

Consider the two linear transformations,

$$T\colon M_{22} \to P_2, \qquad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (2a - b + 3c + d) + (2b - c + 2d)\,x + (4a - 2b + 3c + d)\,x^2$$

$$S\colon P_2 \to \mathbb{C}^2, \qquad S\left(p + qx + rx^2\right) = \begin{bmatrix} 2p + q - 3r \\ 5p + 2q - 4r \end{bmatrix}$$

and the bases of M_{22}, P_2 and $\mathbb{C}^2$ (respectively)

$$B = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 1 & -4 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}, \begin{bmatrix} -2 & -4 \\ -5 & 4 \end{bmatrix} \right\}$$

$$C = \left\{1 + x,\ -2 - 3x + x^2,\ -2 - 2x + x^2\right\}$$

$$D = \left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$$

Verify the conclusion of Theorem MRCLT. In other words, build the three matrix representations of T, S and $S \circ T$ individually and check that they are related by the matrix product as in the theorem.

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R-2 (Section VR)

Let C be the crazy vector space from Section VS (Definition CVS). From Example DC, we know C has dimension 2. By Theorem CFDVS we can conclude that C must be isomorphic to $\mathbb{C}^2$. Construct a function $T\colon \mathbb{C}^2 \to C$ that is a candidate for an isomophism between these two vector spaces by giving an *explicit* formula for T. Then give a convincing argument that T is indeed an isomorphism.

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