Math 290 Name: Dr. Beezer
Exam 2 Fall 2019
Chapter V

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Is the set of vectors $T \subset \mathbb{C}^5$ linearly independent? Justify your answer. (15 points)

$$T = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 4 \\ 4 \\ -8 \end{bmatrix} \right\}$$

2. Is the vector $\mathbf{w}$ an element of the span of $R$, $\langle R \rangle$? Justify your answer. (15 points)

$$\mathbf{w} = \begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix} \qquad T = \left\{ \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ -8 \end{bmatrix} \right\}$$

3. For the following system of linear equations express a typical (generic) solution using the "vector form of the solutions" as described in Theorem VFSLS and related examples. (10 points)

$$\begin{aligned} 3\,x_1 + x_2 + 4\,x_3 - x_4 + 6\,x_5 &= 5 \\ 3\,x_1 - 2\,x_2 - 3\,x_3 + 6\,x_4 - 3\,x_5 &= 4 \\ -x_1 - x_2 - 3\,x_3 + 2\,x_4 - 4\,x_5 &= -2 \end{aligned}$$

4. Find a linearly independent set $R$ so that $\langle R\rangle = \mathcal{N}(A)$, where $\mathcal{N}(A)$ is the null space of $A$. Provide justifications of the requested properties. (15 points)

$$A = \begin{bmatrix} -1 & 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 3 & -1 \\ -2 & 1 & 3 & 4 & 7 \end{bmatrix}$$

5. Suppose $W = \langle S\rangle$. Find a linearly independent set $T$ so that $W = \langle T\rangle$, with justifications of the requested properties. (15 points)

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix} \right\}$$

6. Suppose we have a scalar $\alpha \in \mathbb{C}$ and a vector $\mathbf{x} \in \mathbb{C}^n$. Prove that $\overline{\alpha \mathbf{x}} = \overline{\alpha}\,\overline{\mathbf{x}}$. (15 points)

7. Suppose we have vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w} \in \mathbb{C}^n$ such that $\mathbf{u}$ is orthogonal to $\mathbf{v}$, and $\mathbf{u}$ is orthogonal to $\mathbf{w}$. Prove that $\mathbf{u}$ is orthogonal to $8\mathbf{v} + 3\mathbf{w}$. (15 points)