Math 290 Name: Dr. Beezer
Exam 3 Fall 2019
Chapter M

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices, and note that one problem requires you to not use Sage at all. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Find the solutions of the following system by employing an inverse of the coefficient matrix. **No credit** will be given for solutions obtained by another method. (15 points)

$$\begin{aligned}
-x_1 + 5\,x_2 - 5\,x_3 &= 3\\
-x_1 + 4\,x_2 - 4\,x_3 &= 2\\
3\,x_1 - 4\,x_2 + 5\,x_3 &= 3
\end{aligned}$$

2. Given the matrices A and B below, compute $[AB]_{3,5}$. Do not use Sage in any way to justify your answer, so include enough work to make it clear you did not use Sage. (15 points)

$$A = \begin{bmatrix} -4 & 1 & -1 & 3\\ 6 & -5 & 2 & 9\\ 1 & 4 & 4 & -4\\ -6 & -9 & -2 & 5\\ 9 & -8 & 2 & 7\\ -2 & 9 & 6 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} -7 & -5 & 4 & 1 & -2 & 2 & -4\\ -1 & 3 & 4 & -2 & -4 & -2 & 3\\ 0 & 3 & -2 & -8 & 8 & 0 & -2\\ -3 & -5 & -6 & 2 & 6 & -1 & -4 \end{bmatrix}$$

3. In each part of this question, find a set S of column vectors so that the span of S equals the column space of the matrix A (that is, $\mathcal{C}(A) = \langle C \rangle$) and S satisfies the additional requirements given in each part. Provide clear explanations on why your answer meets all the requirements. (40 points)

$$A = \begin{bmatrix} -5 & -10 & 15 & -17 & -46 & -15 \\ 7 & 14 & -21 & 24 & 65 & 21 \\ 3 & 6 & -9 & 10 & 27 & 9 \\ 2 & 4 & -6 & 7 & 19 & 6 \end{bmatrix}$$

(a) S is a consequence of the **definition** of a column space and requires no computation to create.

(b) S is linearly independent and is a subset of the set of columns of A.

(c) S is linearly independent and is computed using results about row spaces.

(d) S is linearly independent and is computed using the matrix L from the extended echelon form of A.

4. Suppose that A and B are both $m \times n$ matrices. Prove that $A + B = B + A$. (15 points)

5. Suppose that A is an $m \times n$ matrix. Form $M = [A|I_m]$, and then there is a row-equivalent matrix in reduced row-echelon form, $N = [B|J]$ (that is, N is "extended echelon form"). Starting with just this definition of J, give a careful and complete proof that J is nonsingular. (15 points)