Math 290 Name: Dr. Beezer
Exam 4 Fall 2019
Chapter VS

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Determine if the subset T of the vector space of polynomials with degree at most 3, P_3, is linearly independent. (15 points)

   $T = \left\{x^3 - 5x^2 + 4x - 2, x^3 - 5x^2 + 3x - 2, -6x^3 + 4x^2 + x + 7\right\}$

2. Does the set R span the vector space of 2×2 matrices, M_{22}? That is, does $\langle R \rangle = M_{22}$? (15 points)

   $R = \left\{ \begin{bmatrix} 4 & 2 \\ 3 & -5 \end{bmatrix}, \begin{bmatrix} 5 & 3 \\ 4 & -4 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 2 & -8 \end{bmatrix}, \begin{bmatrix} 7 & 8 \\ 6 & 3 \end{bmatrix} \right\}$

3. The set W is a subspace of the vector space of polynomials with degree at most 2, P_2. (You may assume this.) (40 points)

$W = \left\{ a + bx + cx^2 \,\middle|\, a + 2b - c = 0 \right\}$

(a) Prove that the dimension of W is 2, that is, $\dim(W) = 2$.

(b) Does $K = \left\{ 2 + x + 4x^2 \right\}$ span W?

(c) Is $L = \left\{ 5 + 5x^2,\ x + 2x^2,\ 4 + x + 6x^2 \right\}$ a linearly independent subset of W?

(d) Is $B = \left\{ -1 + 3x + 5x^2,\ 3 - 2x - x^2 \right\}$ a basis for W?

4. Prove that U is a subspace of the vector space $\mathbb{C}^3$, by using the three-part test of Theorem TSS. (15 points)

$$U = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle|\, 3a + b + c = 0 \right\}$$

5. Suppose that α, $\beta \in \mathbb{C}$, V is a vector space, $\mathbf{v} \in V$, $\mathbf{v} \neq \mathbf{0}$, and $\alpha\mathbf{v} = \beta\mathbf{v}$. Prove that $\alpha = \beta$. (15 points)