Math 290 Name: Dr. Beezer
Exam 6 Fall 2019
Chapter LT

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Prove that S is a linear transformation. (15 points)

$$S\colon \mathbb{C}^3 \to \mathbb{C}^2, \quad S\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} b+c \\ a+2c \end{bmatrix}$$

2. For the linear transformation S in the previous problem, compute the preimage $S^{-1}\left(\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right)$. (15 points)

3. Consider the linear transformation R from the vector space of 2×2 matrices, M_{22}, to the vector space of polynomials with largest degree 3, P_3. (20 points)

$$R\colon M_{22} \to P_3, \quad R\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (-2a - b - 5c + d) + (-a - b - 4c)x + (2b + 7c + 3d)x^2 + (a + b + 8c + 4d)x^3$$

(a) Compute the kernel of R, $\mathcal{K}(R)$.

(b) Compute the range of R, $\mathcal{R}(R)$.

(c) Is R an invertible linear transformation? Why or why not?

4. Consider the invertible linear transformation T from the vector space $\mathbb{C}^3$ to the vector space of polynomials P_2. Compute an explicit formula for the inverse of T, the linear transformation $T^{-1}\colon P_2 \to \mathbb{C}^3$. (20 points)

$$T\colon \mathbb{C}^3 \to P_2, \quad T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = (-5a - 2b + 2c) + (2a + b - c)x + (-3a - b)x^2$$

5. Suppose that $T\colon U \to V$ and $S\colon V \to W$ are linear transformations. Prove that the composition of S and T, $S \circ T$, is a linear transformation. (15 points)

6. Suppose that $T\colon U \to V$ is a linear transformation which has an inverse **function**, T^{-1}. To prove that T^{-1} is a linear transformation would require checking two defining properties. Choose **one** of the two properties, and prove it. (15 points)