Math 290 Name: Dr. Beezer
Exam 3 Spring 2019
Chapter M

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
For computational problems, place your answer in the provided boxes. Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. For the matrix A below, compute the inverse, or explain how you know A does not have an inverse. (15 points)

$$A = \begin{bmatrix} -1 & 2 & 0 & -2 \\ 0 & 1 & -2 & 2 \\ 0 & -1 & 3 & -4 \\ -1 & 1 & 2 & -5 \end{bmatrix}$$

Answer:

2. A matrix A has the extended echelon form given below. A vector of constants $\mathbf{b}$ is given. Decide if the linear system $\mathcal{LS}(A,\, \mathbf{b})$ is consistent or inconsistent, with a careful explanation. Most of the points for this problem will come from the quality of the explanation. (20 points)

$$\begin{bmatrix} 1 & 0 & -3 & 0 & -1 & 0 & 0 & 4 & -1 \\ 0 & 1 & 1 & 2 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

3. Consider the matrix B. In each part, find a set of vectors whose span is the column space of A, $\mathcal{C}(A)$, and meets the additional requirements, and restrictions on techniques used. Be certain to explain the theorems and definitions employed. (35 points)

$$B = \begin{bmatrix} -7 & 28 & -11 & 19 & 2 & 12 & -8 \\ 11 & -44 & 29 & -65 & -7 & -50 & 32 \\ 4 & -16 & 15 & -37 & -4 & -30 & 19 \\ 1 & -4 & 16 & -46 & -5 & -40 & 25 \\ 0 & 0 & -3 & 9 & 1 & 8 & -5 \end{bmatrix}$$

(a) Use only the definition of a column space.

Answer:

(b) The set is linearly independent and each vector in the set is a column of B.

Answer:

(c) The set is linearly independent and is constructed using theorems about row spaces in non-trivial ways.

Answer:

(d) The set is linearly independent and is constructed using Theorem FS ("Four Subspaces") in a non-trivial way.

Answer:

4. Suppose that A is an $m \times n$ matrix. Prove that $(\alpha A)^t = \alpha A^t$. (15 points)

5. Suppose that Q is an $n \times n$ unitary matrix and $\mathbf{x} \in \mathbb{C}^n$ is any vector. Prove that $\|Q\mathbf{x}\| = \|\mathbf{x}\|$. (This is the statement of a theorem in the book, so do not quote any part of that theorem as part of your answer.) (15 points)