Research Project | Math 491
Dr. R. Beezer | Spring 2020

Research Project As a portion of your work in this year-long advanced course you will carefully research a topic or application in abstract algebra (including group theory), to present in writing and to the class.

Subject Any topic that extends our understanding of abstract algebra, or makes heavy use of abstract algebra in an application, is a legitimate subject. However, you must have your topic approved to avoid duplication with course material or other students' presentations (past or present). Send me an email once you have settled on a topic to "claim" it. If you use a topic that has been done before, your work must be substantially different.

Proposal A one-page proposal is due no later than the end of the day, Sunday, April 5. This should provide evidence that you have a viable subject and are not duplicating course material or another student's subject. A preliminary bibliography of primary sources will provide the best evidence that you have given your subject adequate consideration. These will be returned quickly, and once approved you may schedule your presentation for a time slot during the final two days of the semester.

Written Report Your full written report will be due to me no later than the end of the day, Sunday, April 26. As a rough guideline, I will be expecting 8 to 10 pages (assuming normal margins, fonts and spacing). I will circulate this report to your classsmates prior to your oral presentation. These will also be posted on the course web page, so your report must contain a clear statement of the license you will use, along with your copyright.

Oral Report You will give a 25-minute presentation to the class, accompanied with slides created with LaTeX and Beamer, or PreTeXt and Reveal.js.

Tools Your proposal and written report will be in PDF format, created with LaTeX or PreTeXt (which you can accomplish in SageMathCloud). Your presentation will be accompanied with slides created in LaTeX with the Beamer package or in PreTeXt as a Reveal.js HTML slideshow.

Grading This project represents 10% of your course grade, partitioned as follows: Proposal approved on-time – 1%, Written Report – 3%, Oral Report – 6%. Attendance at your peers' presentations is an integral part of this course, so absences will result in a 1% grade penalty on your own project score.

Date	Component	Format
Sun Apr 5, 11:59 PM	1-page Proposal	LaTeX or PreTeXt PDF via email
Sun Apr 26, 11:59 PM	Written Report	LaTeX or PreTeXt PDF via email
Mon/Tue May 4/5	Class Presentation	Beamer PDF, email in advance

Topic Suggestions

In no particular order. Some topics are very specific, and some of these are very technical. Others are very general and might be approached as providing an introduction, or survey, of the topic. Some have been done by students in prior years. Not all will be appropriate for every student (given different interests and prior coursework).

- Advanced Encryption Standard (AES). Encryption, uses irreducible polynomials, finite fields.
- Orders in Rings. `http://www-math.mit.edu/~kedlaya/Math254B/Orders.pdf`
- Quaternion Algebras. Survey properties, etc.
- Noetherian Rings, Artinian Rings. One, or both.
- Cyclotomic Fields, Cyclotomic Extensions. (Without duplicating course material.)
- Formal Power Series, Laurent Series. One, or both. Laurent Series allow negative-exponent terms.
- Motzkin paper on $\mathbf{Q}[\sqrt{d}]$, `http://projecteuclid.org/handle/euclid.bams/1183514381`
- Composition Algebras. Quaternions, Octonions.
- Resultant of two polynomials, Sylvester Matrix. Related to roots of polynomials.
- Noncommutative Rings. Survey what happens if we drop commutativity, e.g. one-sided ideals.
- Modules. "Vector spaces over rings." (Without duplicating course material from vector space chapter.)
- p-adic Numbers. From an algebraic viewpoint (not analysis). Application to computer arithmetic?
- Algebras. General survey, examples.
- Algebraic Coding Theory. See Judson, Chapters 7 and 20 for a start.
- Burnside's Counting Theorem, Polya's Enumeration Theorem. See Judson, Chapter 12, for a start.
- Finite Groups of Order 16. (This is ambitious, and has been attempted once.)
- Dicyclic Groups.
- Gröbner Bases. (For ideals, see Fraleigh, 7e, Section 28)
- Todd-Coxeter Coset enumeration. (Computer algebra.)
- Classify Finite Rings. (For example, just two rings of order 5.)
- Conway polynomials. (Maybe not a big enough topic.)
- Computational aspects of determining irreducible polynomials.
- Algebraic Graph Theory. (If you concentrate on automorphism groups, not linear algebra.)
- Orders, Filters, Ideals in Ring Theory. (No idea what this is about.)
- Algebraic Number Theory. (Done once in 2010, avoid duplication.)