Math 290A, Monday, March 30 Section B

Mon - B / Sage

Tue - D

Wed - Exam M (Usual time 10:00)

Thu - PD

Fri - Problem Session

Mon - DM / Writing VS

<u>Defn</u> V vector space $B = \{\underset{\sim}{v_1}, \underset{\sim}{v_2}, \ldots, \underset{\sim}{v_m}\} \subseteq V$

Then B is a <u>basis</u> of V if ① B linearly independent

② B spans V

Theorems: BNS, BCS, BRS .... VRRB

Ex $W = \left\langle \left\{ \begin{bmatrix} -4 \\ -3 \\ 3 \\ -11 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \\ -7 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -5 \\ 11 \end{bmatrix} \right\} \right\rangle \subseteq \mathbb{C}^4$

Basis of W? Use theorem BRS. Make vectors <u>rows</u> of a matrix, RREF

$$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

Theorem BRS $\Rightarrow$ B is a basis for W

$$\underset{\sim}{x} = 3\begin{bmatrix} -4 \\ -3 \\ 3 \\ -11 \end{bmatrix} + 1\begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix} + 2\begin{bmatrix} 6 \\ -4 \\ -7 \\ 5 \end{bmatrix} + -2\begin{bmatrix} -1 \\ 0 \\ 3 \\ -4 \end{bmatrix} + 0\begin{bmatrix} -4 \\ 5 \\ -5 \\ 11 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \\ -8 \\ -6 \end{bmatrix} \in W$$

$$\underset{\sim}{x} = 0\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + -10\begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} + -8\begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} = \begin{matrix} 0 \\ -10 \end{matrix} \qquad ?$$

Theorem VRRB V - vector space, $B = \{\underset{\sim}{v}_1, \underset{\sim}{v}_2, \ldots \underset{\sim}{v}_m\}$ basis $\underset{\sim}{v} \in V$

then $\underset{\sim}{v} = a_1 \underset{\sim}{v}_1 + a_2 \underset{\sim}{v}_2 + \cdots + a_m \underset{\sim}{v}_m$ uniquely. (NMUS)

$3 + x^2 - x$

Proof Technique U

Proof

$a_1, a_2, \ldots, a_m$ exist because B spans V.

Suppose
$$\underset{\sim}{v} = a_1 \underset{\sim}{v}_1 + a_2 \underset{\sim}{v}_2 + \cdots + a_m \underset{\sim}{v}_m$$
$$\underset{\sim}{v} = b_1 \underset{\sim}{v}_1 + b_2 \underset{\sim}{v}_2 + \cdots + b_m \underset{\sim}{v}_m$$

$$\underset{\sim}{0} = \underset{\sim}{v} - \underset{\sim}{v} = (a_1 \underset{\sim}{v}_1 + \cdots + a_m \underset{\sim}{v}_m) - (b_1 \underset{\sim}{v}_1 + \cdots + b_m \underset{\sim}{v}_m)$$ vector space properties

$$= (a_1 - b_1)\underset{\sim}{v}_1 + (a_2 - b_2)\underset{\sim}{v}_2 + \cdots + (a_m - b_m)\underset{\sim}{v}_m$$ $\Leftarrow$ RLD on a LI set

$\Rightarrow$ $a_1 - b_1 = 0$ $a_2 - b_2 = 0$ .... $a_m - b_m = 0$

$a_1 = b_1$ $a_2 = b_2$ $a_m = b_m$