

Math 290 A, Thursday, April 23 Section IVLT

Fri - Problem Session

Mon - VR (RQ)
- Writing

Tue - MR

Wed - Exam LT

Wed May 15 Noon - Final Exam?

Defn $T: U \rightarrow V$ is invertible if there exists $S: V \rightarrow U$

so that (1) $(S \circ T)(\underline{u}) = \underline{I}_U(\underline{u}) = \underline{u}$ (2) $(T \circ S)(\underline{v}) = \underline{I}_V(\underline{v}) = \underline{v}$

we call S, T^{-1} .

$$S \circ T = I_U$$

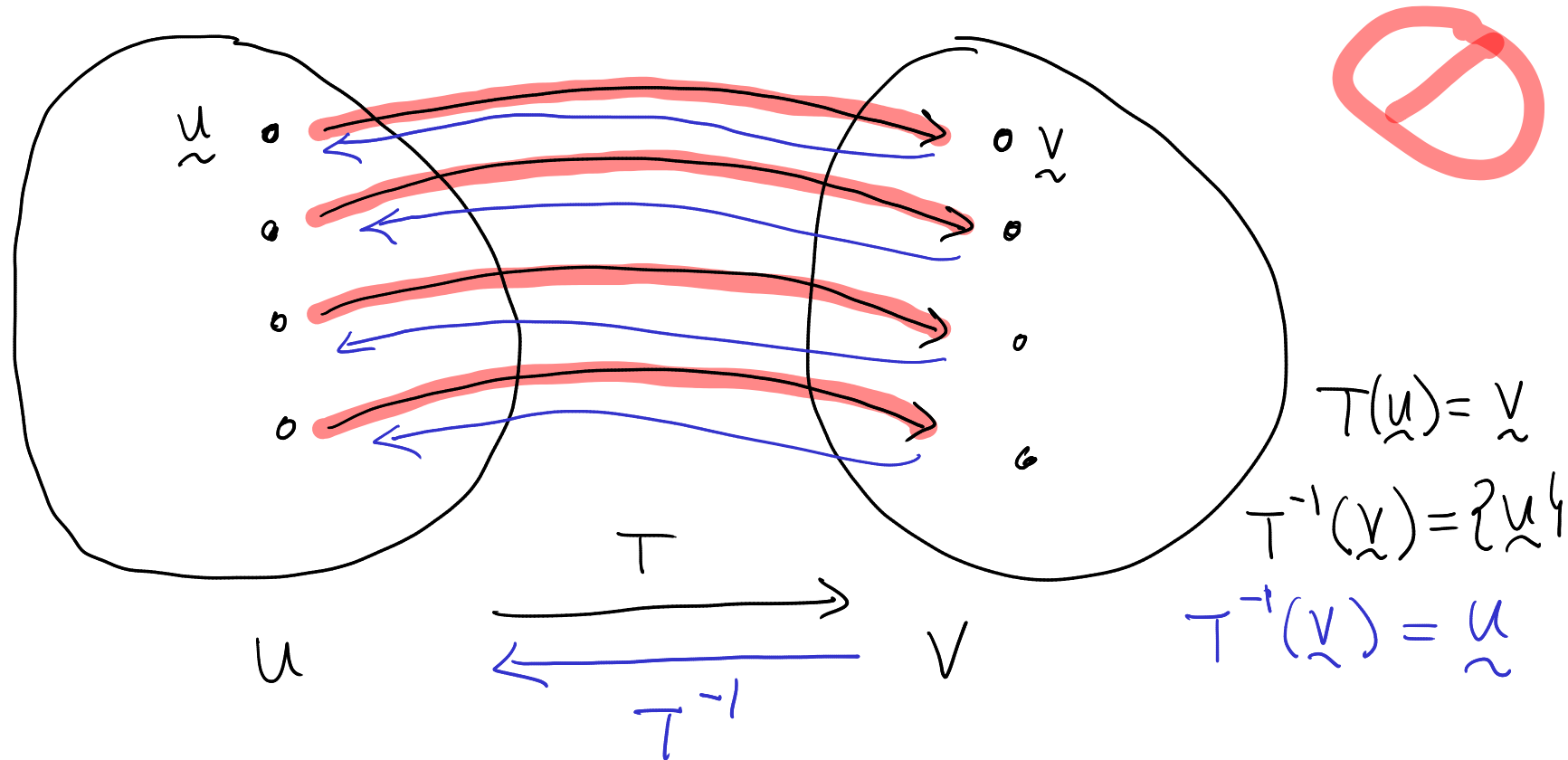
$$T \circ S = I_V$$

Theorem ILTIS T invertible $\iff T$ injective & T surjective

Proof (\Leftarrow)

T is injective & surjective

Blue arrow
is
 $T^{-1}: V \rightarrow U$



\underline{Ex} $T: P_3 \rightarrow M_{22}$ $T(ax^2+bx+c+dx^3) = \begin{bmatrix} a+b+c+d & 2a+3b+4c-d \\ -a-b+d & 2a+3b+5c+2d \end{bmatrix}$

$K(T) = \{0+0x+0x^2+0x^3\} \Rightarrow n(T) = 0 \Rightarrow T$ injective

$R(T) = M_{22} \Rightarrow r(T) = 4 = \dim(M_{22}) \Rightarrow T$ surjective

ILTS $\Rightarrow T$ is invertible ; $T^{-1}???$ Start w/ basis of codomain.

$C = \{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \}$ Preimages of basis elements

$T^{-1}(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}) : T(ax^2+bx+c+dx^3) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} a+b+c+d & 2a+3b+4c-d \\ -a-b+d & 2a+3b+5c+2d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ system RREF \rightarrow

$\begin{bmatrix} 1 & 0 & 0 & 0 & | & 10 \\ 0 & 1 & 0 & 0 & | & -11 \\ 0 & 0 & 1 & 0 & | & 3 \\ 0 & 0 & 0 & 1 & | & 7 \end{bmatrix}$

$T^{-1}(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}) : \quad \quad \quad = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \quad \quad \quad = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ system RREF \rightarrow

$\begin{bmatrix} 1 & 0 & 0 & 0 & | & 5 \\ 0 & 1 & 0 & 0 & | & -6 \\ 0 & 0 & 1 & 0 & | & 2 \\ 0 & 0 & 0 & 1 & | & -1 \end{bmatrix}$

$T^{-1}(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix})$ \rightarrow $\begin{bmatrix} I_4 & | & \begin{matrix} 7 \\ -9 \\ 3 \\ -1 \end{matrix} \end{bmatrix}$

$T^{-1}(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix})$ \rightarrow $\begin{bmatrix} I_4 & | & \begin{matrix} 6 \\ 7 \\ -2 \\ 1 \end{matrix} \end{bmatrix}$

"Like" ANM \rightarrow



$$T^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \{10 - 11x + 3x^2 - x^3\}$$

$$T^{-1}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \{5 - 6x + 2x^2 - x^3\}$$

$$T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \{7 - 9x + 3x^2 - x^3\}$$

$$T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \{-6 + 7x - 2x^2 + x^3\}$$

← all four preimages are singulars

$$T^{-1}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = T^{-1}\left(a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + d \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

(VRPB) → (LTLC)

$$= a T^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) + b T^{-1}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) + c T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) + d T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$= a(10 - 11x + 3x^2 - x^3) + b(5 - 6x + 2x^2 - x^3) + c(7 - 9x + 3x^2 - x^3) + d(-6 + 7x - 2x^2 + x^3)$$

$$= (10a + 5b + 7c - 6d) + (-11a - 6b - 9c + 7d)x + (3a + 2b + 3c - 2d)x^2 + (-a - b - c + d)x^3$$

$$T^{-1}: M_{22} \rightarrow P_3$$