Math 290A, Friday, May 1 Problem Session

Mon - CB (RQ)

- Sage

Tue - Problem Session / Housekeeping

Writing 10 AM

Wed - Exam R

Final Exam

Tuesday, May 12

9 AM - two hour design / three hour limit

(8 AM by appointment)

Ex $M_{2,3}$ & $\mathbb{C}^6$ $\dim(M_{2,3}) = 6 = \dim(\mathbb{C}^6) \implies M_{2,3} \cong \mathbb{C}^6$

$\rho_B : M_{2,3} \to \mathbb{C}^6$

invertible

$$B = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right\}$$

Coordinatization principle $\implies$ B basis

$\rho_C \downarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$C^* = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ldots \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

So ρ_B is an isomorphism

Ex $M_{2,3}$ & P_5 $E = \{1, x, x^2, x^3, x^4, x^5\}$ basis of P_5

$$\begin{tikzcd} M_{2,3} \arrow[r, "\rho_B"'] \arrow[rr, bend right] & \mathbb{C}^6 & P_5 \arrow[l, "\rho_E"] \end{tikzcd}$$

invertible $\to$ $\rho_E^{-1} \circ \rho_B : M_{2,3} \to P_5$

VR. M10

M_{22}

$$S = \left\{ \begin{bmatrix} 33 & 99 \\ 78 & -9 \end{bmatrix}, \begin{bmatrix} -16 & -47 \\ -36 & 2 \end{bmatrix}, \begin{bmatrix} 10 & 27 \\ 17 & 3 \end{bmatrix}, \begin{bmatrix} -2 & -7 \\ -6 & 4 \end{bmatrix} \right\}$$ basis?

$$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \quad \left(B' = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\} \right)$$

$$T = \left\{ \rho_B\left(\begin{bmatrix} 33 & 99 \\ 78 & -9 \end{bmatrix} \right), \dots \right\} = \left\{ \begin{bmatrix} 33 \\ 99 \\ 78 \\ -9 \end{bmatrix}, \begin{bmatrix} -16 \\ -47 \\ -36 \\ 2 \end{bmatrix}, \begin{bmatrix} 10 \\ 27 \\ 17 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ -7 \\ -6 \\ 4 \end{bmatrix} \right\} \subseteq \mathbb{C}^4$$

Is T linearly independent?

$$E = \begin{bmatrix} 33 & | & | & | \\ 99 & | & | & | \\ -78 & | & | & | \\ -9 & | & | & | \end{bmatrix} \xrightarrow{\text{RREF}} I_4$$

$r = 4 = n$

$\Rightarrow T$ linearly independent

LIVRN

E row reduces to $I_n \Rightarrow E$ nonsingular $\Rightarrow$ columns of E are basis of $\mathbb{C}^4$

NME Part 8

$\Rightarrow T$ basis of $\mathbb{C}^4$ $\Rightarrow$ S basis of M_{22}

Theorem CLI
Theorem CSS

Coordinatization Principle