Math 290 A, Monday, May 4, Section CB

Mon - CB

Tue - Problem Session ← House keeping

- Writing ~~R~~ (10 AM)

Wed - Exam R

Tue, May 12 Final Exam

9 - 12 Pacific

8 (by appointment)

Change-of-Basis

Matrix representation of identity L.T.

$I_V : V \to V \qquad I_V(\underset{\sim}{v}) = \underset{\sim}{v}$

B, C bases of V

$M^{I_V}_{B,C} = C_{B,C}$

<u>Fact</u> $\rho_C(I_V(\underset{\sim}{v})) = M^{I_V}_{B,C}\, \rho_B(\underset{\sim}{v})$ FTMR

$\underline{\rho_C(\underset{\sim}{v})} = C_{B,C}\, \underline{\rho_B(\underset{\sim}{v})}$

Ex M_{22} $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$C = \left\{ \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix} \right\}$

$$C_{C,B} = \begin{bmatrix} 1 & 2 & 1 & 2 \\ -2 & -3 & -3 & -2 \\ 1 & 3 & 1 & 4 \\ -1 & 0 & -2 & 3 \end{bmatrix} \qquad C_{B,C} = C_{C,B}^{-1} = \begin{bmatrix} & & & \\ & & & \end{bmatrix}^{-1} = \begin{bmatrix} 6 & -1 & -5 & 2 \\ -1 & 2 & 3 & -2 \\ -3 & -1 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix}$$

MR: Catch-up T invertible $M^{T^{-1}}_{B,C} = \left(M^{T}_{C,B}\right)^{-1}$ (see Exercise MR.C40)

$T: U \to V$ B, C bases of U ; D & E bases of V

$$M^{T}_{B,D} = C_{E,D}\, M^{T}_{C,E}\, C_{B,C}$$

$$\rho_D(T(\underset{\sim}{u})) \leftarrow \rho_B(\underset{\sim}{u}) \quad \Big\| \quad \rho_D(T(\underset{\sim}{u})) \leftarrow \rho_E(T(\underset{\sim}{u})) \leftarrow \rho_C(\underset{\sim}{u}) \longleftarrow \rho_B(\underset{\sim}{u})$$

Ex $\quad D = \{1, x, x^2\}, \quad E = \{1, 1+x, 1+x+x^2\}$

$$\underbrace{\overbrace{\begin{bmatrix} -5 & -12 & -5 & -12 \\ 10 & 20 & 12 & 16 \\ 0 & 2 & -1 & 4 \end{bmatrix}}^{3\times 4}}_{M^T_{C,E}} = \underbrace{\overbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}^{3\times 3}}_{\substack{C_{D,E} \\ (C^{-1}_{E,D})}} \underbrace{\overbrace{\begin{bmatrix} 2 & 1 & 3 & -2 \\ 5 & 3 & 7 & -4 \\ 1 & 1 & 1 & 0 \end{bmatrix}}^{3\times 4}}_{M^T_{B,D}} \underbrace{\overbrace{\begin{bmatrix} 1 & 2 & 1 & 2 \\ -2 & -3 & -3 & -2 \\ 1 & 3 & 1 & 4 \\ -1 & 0 & -2 & 3 \end{bmatrix}}^{4\times 4}}_{C_{C,B}}$$

Specialize $\quad T: U \to U \quad B = D \quad C = E \quad$ (matrix representations square)

$$M^T_{B,B} = C_{C,B}\, M^T_{C,C}\, C_{B,C}$$

$$= C^{-1}_{B,C}\, M^T_{C,C}\, C_{B,C} \qquad \text{similarity } \underline{\underline{\text{is}}} \text{ change-of-basis}$$

<u>Defn</u> $T: V \to V$ then $\underset{\sim}{v} \neq \underset{\sim}{0}$ is an <u>eigenvector</u> of T if $T(\underset{\sim}{v}) = \lambda \underset{\sim}{v}$

Diagonalization: $T: V \to V$

1. Build an <u>easy</u> matrix representation (nice basis) B
2. Compute eigenvalues, eigenvectors (column vectors)
3. Un coordinatize the eigenvectors (ρ_B^{-1}, results in V)
4. New basis C = these eigenvectors of V
5. Matrix representation of T relative to C

(Notes: <u>First</u> M.R.)