

Math 290 A, Monday, May 4,

Mon - CB

Tue - Problem Session  $\leftarrow$  House keeping

- Writing ~~R~~ (10 AM)

Wed - Exam R

Thu, May 12

Final Exam

9-12 Pacific

8 (by appointment)

Section CB

Change-of-Basis

Matrix representation of identity  $I_V$ .

$$I_V : V \rightarrow V \quad I_V(\underline{v}) = \underline{v}$$

$B, C$  bases of  $V$

$$M_{B,C}^{I_V} = C_{B,C}$$

$$\underline{\text{Fact}} \quad \rho_C(I_V(\underline{v})) = M_{B,C}^{I_V} \rho_B(\underline{v}) \quad \text{FTMR}$$

$$\underline{\rho_C(\underline{v})} = C_{B,C} \underline{\rho_B(\underline{v})}$$

Ex  $M_{22}$   $B = \{ [1\ 0], [0\ 1], [0\ 0], [0\ 0] \}$

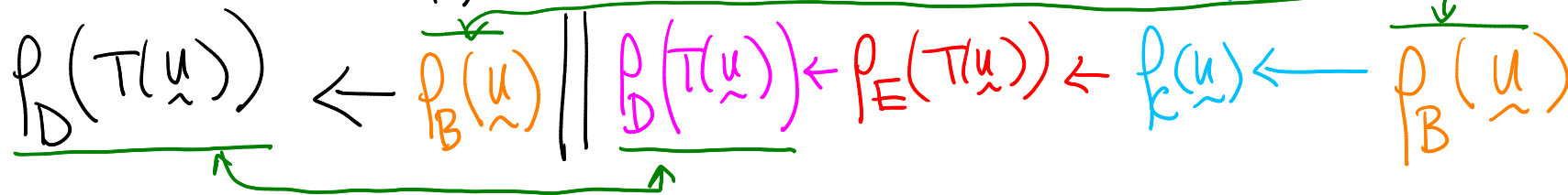
$C = \{ [1\ -2], [2\ -3], [1\ -3], [2\ -2] \}$

$C_{C,B} = \begin{bmatrix} 1 & 2 & 1 & 2 \\ -2 & -3 & -3 & -2 \\ 1 & 3 & 1 & 4 \\ -1 & 0 & -2 & 3 \end{bmatrix}$   $C_{B,C} = C_{C,B}^{-1} = \begin{bmatrix} 6 & -1 & -5 & 2 \\ -1 & 2 & 3 & -2 \\ -3 & -1 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix}$

MR: Catch-up  $T$  invertible  $M_{B,C}^T = (M_{C,B})^{-1}$  (see Exercise MR.C40)

$T: U \rightarrow V$   $B, C$  bases of  $U$ ;  $D, E$  bases of  $V$

$M_{B,D}^T = C_{E,D} M_{C,E}^T C_{B,C}$



Ex

$$D = \{1, X, X^2\}, \quad E = \{1, 1+X, 1+X+X^2\}$$

$$\begin{bmatrix} -5 & -12 & -5 & -12 \\ 10 & 20 & 12 & 16 \\ 0 & 2 & -1 & 4 \end{bmatrix}_{3 \times 4} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 2 & 1 & 3 & -2 \\ 5 & 3 & 7 & -4 \\ 1 & 1 & 1 & 0 \end{bmatrix}_{3 \times 4} \begin{bmatrix} 1 & 2 & 1 & 2 \\ -2 & -3 & -3 & -2 \\ -1 & 3 & 1 & 4 \\ -1 & 0 & -2 & 3 \end{bmatrix}_{4 \times 4}$$

$$\uparrow \\ M_{C,E}^T$$

$$C_{D,E}$$

$$\uparrow \\ M_{B,D}^T$$

$$\uparrow \\ C_{C,B}$$

$$(C_{E,D}^{-1})$$

Speicherze

$$T: U \rightarrow U$$

$$B=D$$

$$C=E$$

(matrix representation square)

$$M_{B,B}^T = C_{C,B} M_{C,C}^T C_{B,C}$$

$$= C_{B,C}^{-1} M_{C,C}^T C_{B,C}$$

similarity  $\equiv$  change-of-basis

Defn  $T: V \rightarrow V$  then  $\underline{v} \neq \underline{0}$  is an eigenvector of  $T$   
if  $T(\underline{v}) = \lambda \underline{v}$

Diagonalization:  $T: V \rightarrow V$

① Build an easy matrix representation (nice basis)  $\leftarrow B$

② Compute eigenvalues, eigenvectors (column vectors)

③ Un-coordinate the eigenvectors  $\leftarrow (P_B^{-1}, \text{result in } V)$

④ New basis  $C =$  these eigenvectors  $\leftarrow$  of  $V$

⑤ Matrix representation of  $T$  relative to  $C$

(Notes: First M.R.)