Math 290B, Thursday, April 2 Section PD

Fri – Section DM (RQ)

Mon – Problem Session
Writing VS

<u>Defn</u> V – vector space, $B = \{\underset{\sim}{v_1}, \underset{\sim}{v_2}, \ldots, \underset{\sim}{v_m}\} \subseteq V$ then the <u>dimension</u> of V is m, write $\dim(V) = m$ (\dim(V))

"<u>well-defined</u>"

Ex $A = \begin{bmatrix} 1 & 2 & -1 & -6 \\ 0 & 1 & -2 & -2 \\ 1 & 1 & 1 & -4 \end{bmatrix}$ $N(A)$? $\xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Basis from Theorem BNS $B = \left\{ \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ Others

$\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 14 \\ 14 \\ 0 \\ 7 \end{bmatrix}$ Mira S. / $\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 0 \\ 2 \end{bmatrix}$ Maggie H. / $\begin{bmatrix} -153 \\ 102 \\ 51 \\ 0 \end{bmatrix}, \begin{bmatrix} 136 \\ 136 \\ 0 \\ 68 \end{bmatrix}$ Jasmine M.

$\begin{bmatrix} 8 \\ 0 \\ -8/5 \\ 8/5 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 8/5 \\ 24/10 \end{bmatrix}$ Anon. / $\begin{bmatrix} -1 \\ 2/3 \\ 1/3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2/3 \\ 2/3 \\ 0 \\ 1/3 \end{bmatrix}$ KatieR /

$\left\{ 4 \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} + 6 \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

Theorem BIS V - vector space, B & C bases $\Rightarrow |B| = |C|$

<u>Proof</u> Suppose $|B| > |C|$, C spans V so SSLD $\Rightarrow B$ linearly dependent $\not\Leftarrow\Rightarrow$

Suppose $|C| > |B|$, B spans V, so SSLD $\Rightarrow C$ linearly dependent $\not\Leftarrow\Rightarrow$

from HMVEI

We are left $|B| = |C|$.

Rank & Nullity

$r = \dim(C(A)) = \text{rank}$

Section PD

Theorem G V - vector space, $\dim(V) = m$

1) $|C| > m \Rightarrow C$ linearly dependent
2) $|C| < m \Rightarrow C$ does not span V
3) $|C| = m$ + C spans $V \Rightarrow C$ linearly independent
4) $|C| = m$ + C linearly independent $\Rightarrow C$ spans V

Proof Uses SSLD!

Ex M_{22} $\dim(M_{22}) = 2 \cdot 2 = 4$ $X = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a + 2b - c + 3d = 0 \right\}$ subspace

spanning set for X?

$X = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a = -2b + c - 3d \right\} = \left\{ \begin{bmatrix} -2b+c-3d & b \\ c & d \end{bmatrix} \middle| b, c, d \in \mathbb{C} \right\}$

$= \left\{ b \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} + c \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + d \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix} \middle| b, c, d \in \mathbb{C} \right\} = \left\langle \left\{ \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix} \right\} \right\rangle$

$S = \left\{ \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ spans X, S is linearly independent.

So S is a basis for X, $\dim(X) = 3$.

$R = \left\{ \begin{bmatrix} -7 & -2 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} -18 & 3 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} -19 & 4 \\ 1 & 4 \end{bmatrix} \right\} \subseteq X$ Claim R is a basis of X.

$$a_1 \begin{bmatrix} -7 & -2 \\ 1 & 4 \end{bmatrix} + a_2 \begin{bmatrix} -18 & 3 \\ 3 & 5 \end{bmatrix} + a_3 \begin{bmatrix} -19 & 4 \\ 1 & 4 \end{bmatrix} = \underset{\sim}{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} -7a_1 - 18a_2 - 19a_3 &= 0 \\ -2a_1 + 3a_2 + 4a_3 &= 0 \\ a_1 + 3a_2 + a_3 &= 0 \\ 4a_1 + 5a_2 + 4a_3 &= 0 \end{aligned}$$

4 equations, 3 variables

homogenous

$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

So $a_1 = a_2 = a_3 = 0$

So R is linearly independent

Theorem G (4) $\Rightarrow$

R spans X.

$\to -7 + 2(-2) - 1 + 3(4) = 0$ so $\in X$

(two more)