Math 290 B, Tuesday, April 14, Problem Session

Wed - Exam D&E

Thu - LT (RQ!)

Fri - ILT

W/WF, CR/NC

Sage

A.det( ) (determinant)

A.fcp( )

(RREF)

& "by hand"

PEE. T22 $\quad U$ unitary, λ eigenvalue $\Rightarrow |\lambda| = 1$

$\lambda = a+bi$

$|\lambda| = \sqrt{a^2+b^2}$

$= \sqrt{\lambda \bar{\lambda}}$ Theorem

Proof

$UU^* = I \Rightarrow U^*U = I$ (U & U^* inverses)

① $\langle U\underset{\sim}{x}, U\underset{\sim}{x} \rangle = \langle U^*U\underset{\sim}{x}, \underset{\sim}{x} \rangle = \langle I\underset{\sim}{x}, \underset{\sim}{x} \rangle = \langle \underset{\sim}{x}, \underset{\sim}{x} \rangle$ (UMPIP)

② $\langle U\underset{\sim}{x}, U\underset{\sim}{x} \rangle = \langle \lambda\underset{\sim}{x}, \lambda\underset{\sim}{x} \rangle = \bar{\lambda} \langle \underset{\sim}{x}, \lambda\underset{\sim}{x} \rangle = \bar{\lambda}\lambda \langle \underset{\sim}{x}, \underset{\sim}{x} \rangle$

Let $\underset{\sim}{x}$ be an eigenvector of A for λ

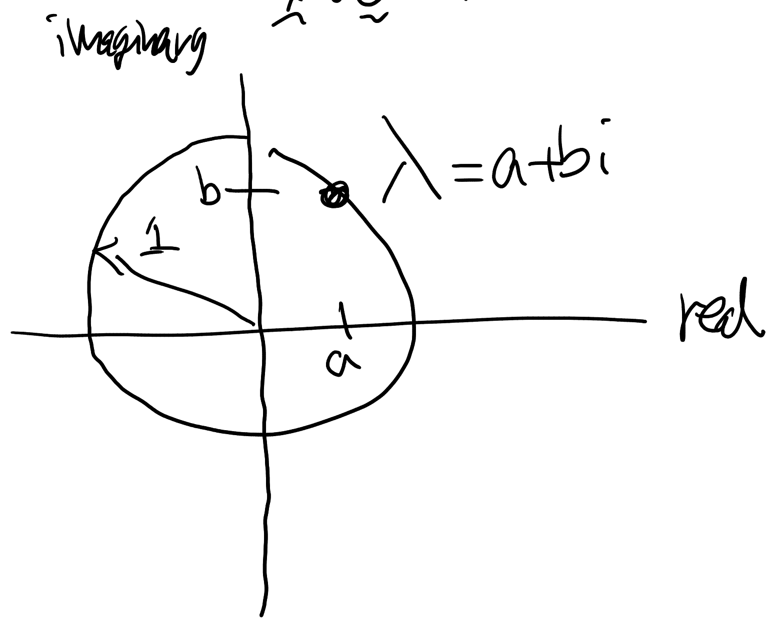
So ① + ② $\quad \cancel{\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle} = \bar{\lambda}\lambda \cancel{\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle}$

$1 = \bar{\lambda}\lambda$

$\sqrt{1} = \sqrt{\bar{\lambda}\lambda}$

$1 = |\lambda|$

$\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle \neq 0$ since $\underset{\sim}{x} \neq \underset{\sim}{0}$ & Theorem PIP



PEE.T21 A has eigenvalue λ For $\underline{\alpha \in \mathbb{C}}$

$\Rightarrow$ $\lambda + \alpha$ eigenvalue of $A + \alpha I$

(a) Grab $\underset{\sim}{x}$, an eigenvector of A for λ. Then

$$(A + \alpha I)\underset{\sim}{x} = A\underset{\sim}{x} + \alpha I \underset{\sim}{x} = \lambda \underset{\sim}{x} + \alpha \underset{\sim}{x} = (\lambda + \alpha)\underset{\sim}{x}$$

So $\lambda + \alpha$ eigenvalue of $A + \alpha I$.

(b)
$$p_{A+\alpha I}(x) = \det(A + \alpha I - xI)$$
$$= \det(A + (\alpha - x) I)$$

$$\underline{p_{A+\alpha I}(\lambda+\alpha)} = \det(A + (\alpha - (\lambda+\alpha)) I)$$
$$= \det(A + (-\lambda) I)$$
$$= \det(A - \lambda I) = p_A(\lambda) = \underline{0}$$