Math 290B, Friday, April 24 Problem Session

Mon - VR (RQ)

Writing 11 AM

Tue - MR

Wed - Exam LT

$\underline{U \ \& \ V}$ are $\underline{\text{isomorphic}}$ if ...., write $U \cong V$

vector spaces

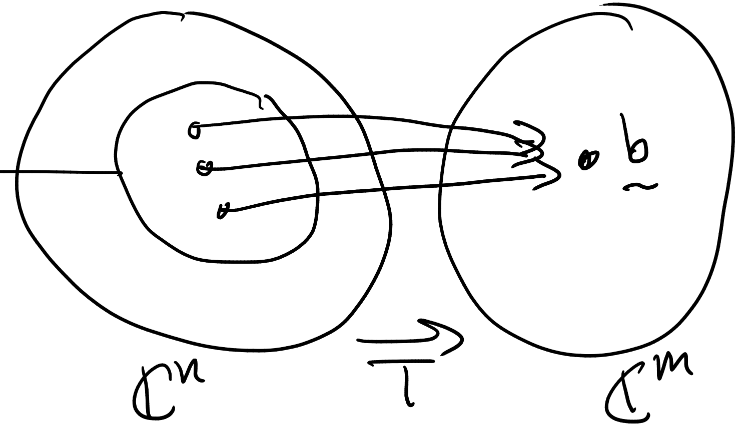
T is an $\underline{\text{isomorphism}}$ (invertible linear transformation)

$LS(A, \underset{\sim}{b})$ systems of linear equations (SLEMM: $A\underset{\sim}{x} = \underset{\sim}{b}$)

LT: $T : \mathbb{C}^n \to \mathbb{C}^m$, $T(\underset{\sim}{x}) = A\underset{\sim}{x}$

Solutions: $T^{-1}(\underset{\sim}{b})$

solution set



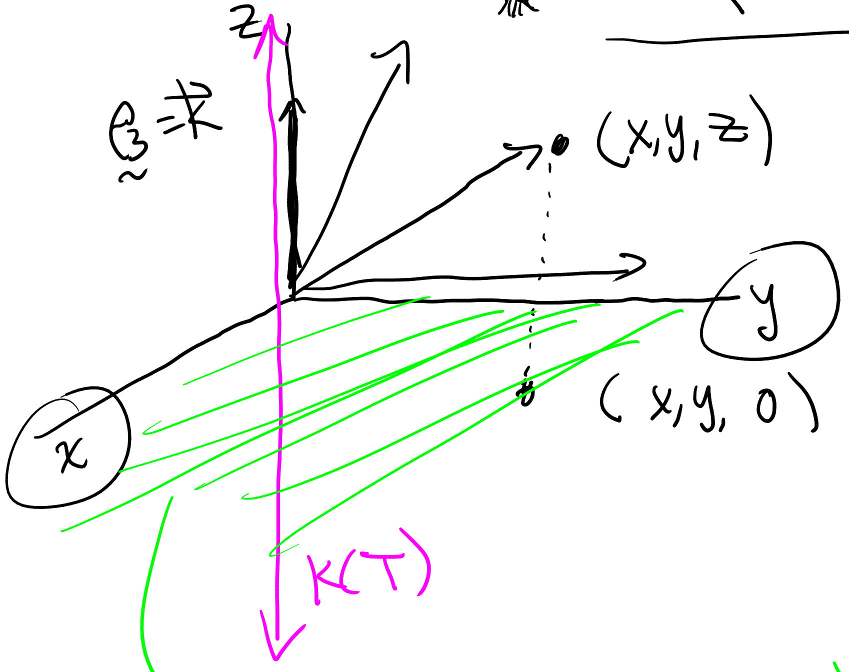
homogeneous

$\underline{\text{Solutions to } LS(A, \underset{\sim}{0})}$

$N(A) = T^{-1}(\underset{\sim}{0}) = K(T)$

IVLT. C25 (b)

$T: \mathbb{C}^3 \to \mathbb{C}^3 \quad \mid \quad T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$ "projection"



x-y plane $= R(T)$

$T(\underset{\sim}{e_1}) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$T(\underset{\sim}{e_2}) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$T(\underset{\sim}{e_3}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$K(T) = \left\{ \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} \middle| \, z \in \mathbb{C} \right\} = \langle \{ \underset{\sim}{e_3} \} \rangle$

"orthogonal"

$n(T) = 1$

$R(T) = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \middle| \, x, y \in \mathbb{C} \right\} = \langle \{ \underset{\sim}{e_1}, \underset{\sim}{e_2} \} \rangle$

$r(T) = 2$

$r(T) + n(T) = \dim(\mathbb{C}^3) = 3$

domain

IVLT.T40 $T: U \to V$ $\quad \dim(U) = \dim(V) = \dim(W)$ $\quad \Rightarrow \quad T$ invertible

$S: V \to W$ $\quad S \circ T$ invertible $\quad$ & S invertible

<u>Proof</u>

$K(T) \subseteq K(S \circ T)$ (ILT.T15) $\quad \underset{\sim}{u} \in K(T) \Rightarrow T(\underset{\sim}{u}) = \underset{\sim}{0}$

$= \{\underset{\sim}{0}\}$ $\quad S \circ T$ injective KILT $\quad (S \circ T)(\underset{\sim}{u}) = S(T(\underset{\sim}{u})) = S(\underset{\sim}{0}) = \underset{\sim}{0}$

$\Rightarrow \underset{\sim}{u} \in K(S \circ T)$

$\Rightarrow K(T) = \{\underset{\sim}{0}\} \Rightarrow T$ injective $\Rightarrow n(T) = 0$

KILT

$r(T) = \underbrace{r(T) + n(T)}_{} \underbrace{- n(T) + n(T)}_{\text{zero}}$ $\; - n(T)$

$= \dim(U) - n(T)$

$= \dim(V) - 0$

$= \dim(V)$

always

$R(T) \subseteq V \Rightarrow R(T) = V \Rightarrow T$ surjective

equal dimensions