Math 290 B, Tuesday, May 5 Problem session

Wed - Exam R

- matrix representations
- Sage RREF
  - Determinants
  - Eigen-stuff

- Comprehensive
- Writing R (?)

Fri - Group office Hours

May 8 10-Noon Pacific

Final Exam

- Tuesday, May 12
- Two-hour design (6/7 pages)
- Three-hour limit
- 9-12 Pacific
- 8-11 by appointment

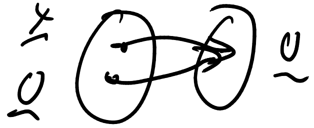
CB. T15

$T: V \Rightarrow V$ $\quad$ T injective iff $\lambda = 0$ not an eigenvalue of T.

($\Rightarrow$) $\quad T$ injective $\Rightarrow \lambda = 0$ not eigenvalue

Contrapositive: $\quad \lambda = 0$ is an eigenvalue $\Rightarrow T$ not injective

LTTZZ



There is $\underset{\sim}{x} \neq \underset{\sim}{0}$, $T(\underset{\sim}{x}) = 0\underset{\sim}{x}$ $\quad$ So $T(\underset{\sim}{x}) = \underset{\sim}{0} = T(\underset{\sim}{0})$

$= \underset{\sim}{0}$ $\quad$ $\underset{\sim}{x} \neq \underset{\sim}{0}$ $\quad$ thus T not injective

($\Leftarrow$) $\quad \lambda = 0$ not an eigenvalue $\Rightarrow T$ injective

Contrapositive: $\quad T$ not injective $\Rightarrow \lambda = 0$ is an eigenvalue

So there is $\underset{\sim}{x}, \underset{\sim}{y}$, $\underset{\sim}{x} \neq \underset{\sim}{y}$ with $T(\underset{\sim}{x}) = T(\underset{\sim}{y})$

$$T(\underset{\sim}{x} - \underset{\sim}{y}) = T(\underset{\sim}{x}) - T(\underset{\sim}{y}) = \underset{\sim}{0} = 0\,(\underset{\sim}{x} - \underset{\sim}{y})$$

T L.T.

Note $\underset{\sim}{x} \neq \underset{\sim}{y}$

$\underset{\sim}{x} - \underset{\sim}{y} \neq \underset{\sim}{0}$

So $\lambda = 0$ is an eigenvalue of T. (for $\underset{\sim}{x} - \underset{\sim}{y}$).

Spring 2016 Exam R, #5 (CB.C40) L^{-1}???

$L: \mathbb{C}^3 \to P_2$ $L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = (3a-b+3c)+(4a-b+3c)x+(4a+2b-7c)x^2$

$B = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$ $C = \{1, x, x^2\}$

$M^{L}_{B,C} = \begin{bmatrix} 3 & -1 & 3 \\ 4 & -1 & 3 \\ 4 & 2 & -7 \end{bmatrix}$

"on sight"

$M^{L^{-1}}_{C,B} = (M^{L}_{B,C})^{-1} = [\ \ ]^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ -40 & 33 & -3 \\ -12 & 10 & -1 \end{bmatrix}$

$L^{-1}(a+bx+cx^2) = \rho_B^{-1}\left(M^{L^{-1}}_{C,B}\,\rho_C(a+bx+cx^2)\right) = \rho_B^{-1}\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right)$

FTMR

$= \rho_B^{-1}\left(\begin{bmatrix} -a+b \\ -40a+33b-3c \\ -12a+10b-c \end{bmatrix}\right) = \begin{bmatrix} -a+b \\ -40a+33b-3c \\ -12a+10b-c \end{bmatrix}$