Math 290 Name: Dr. Beezer
Exam 2 Spring 2021
Chapter V

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Is the set R linearly independent or not? Give complete reasons for why your answer is correct. (15 points)

$$R = \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 1 \\ 4 \\ 2 \end{bmatrix} \right\}$$

2. Is $\mathbf{u}$ in the span of U? In other words is $\mathbf{u} \in \langle U \rangle$? Give complete reasons for why your answer is correct. (20 points)

$$\mathbf{u} = \begin{bmatrix} 5 \\ 1 \\ 2 \\ 1 \end{bmatrix} \qquad U = \left\{ \begin{bmatrix} -1 \\ 2 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \\ 0 \end{bmatrix} \right\}$$

3. Determine a set S that is linear independent, and whose span is the null space of A. That is, $\mathcal{N}(A) = \langle S \rangle$. Give complete reasons for why your answer is correct. (15 points)

$$A = \begin{bmatrix} 1 & 0 & -1 & -5 & 1 \\ -2 & 1 & 0 & 6 & 1 \\ 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 6 & -4 \end{bmatrix}$$

4. Determine a set T that is linear independent, and whose span is equal to the span of S. That is, $\langle T \rangle = \langle S \rangle$. Give complete reasons for why your answer is correct. (15 points)

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ -4 \\ 1 \end{bmatrix} \right\}$$

5. Prove that for any $\mathbf{u} \in \mathbb{C}^m$, $1\mathbf{u} = \mathbf{u}$. (15 points)

6. Suppose that $\mathbf{u}$, $\mathbf{v} \in \mathbb{C}^m$ are orthogonal vectors with equal norms. Prove that $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are orthogonal vectors. (20 points)