Class—FCLA CP

Advanced Linear Algebra

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Eigenvalues are hard. Everything else is easy.

Following is verbatim from an FCLA/290 in-class worksheet.

A totally random matrix is unlikely to have a characteristic polynomial that factors if we restrict ourseves to the rationals. But we can find all the roots over $\overline{Q}$, the set of all algebraic numbers. (This is the set of all real roots of all possible polynomials with integer coefficients.)

```
D = random_matrix(QQ, 10)
D
```

```
p = D.characteristic_polynomial()
p.factor()
```

```
p.roots(ring=QQbar, multiplicities=False)
```

If we make a "block diagonal" matrix, then the characteristic polynomial will definitely factor some

```
E = block_diagonal_matrix( [random_matrix(QQ, 5),
    random_matrix(QQ, 5)])
E
```

```
p = E.charpoly()
p.factor()
```

Finally a large example, illustrating how fast Sage is at making characteristic polynomials and at factoring.

```
F = block_diagonal_matrix( [random_matrix(QQ, 50),
    random_matrix(QQ, 50)])
p = F.charpoly()
p.factor()
```

This is such a common operation, that Sage has a shorthand method for the **Factored Characteristic Polynomial**, namely `.fcp()`.

```
F.fcp()
```

Nothing short of amazing!