

# Class—FCLA MP

## Advanced Linear Algebra

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Rational Canonical Form is like Jordan Canonical Form, but does not require the eigenvalues of the matrix. And all of the entries of RCF are from the same field as the original matrix.

We begin with a smaller matrix, and find the minimal polynomial.

```
A = matrix(QQ, [
[-22, -20, 43, -64, -23],
[ 17,  7, -33, 41, 17],
[-17, -20, 33, -55, -18],
[-16,  0, 31, -30, -15],
[ 27, -20, -53, 29, 24]
])
A
```

A polynomial generator is convenient.

```
x = polygen(QQ, 'x')
```

Similar to the polynomial for determining eigenvalues, but we repeat for a full basis, and then take the least common multiple.

```
e = vector(QQ, [1,0,0,0,0])
K = column_matrix([A^i*e for i in
    range(6)]).right_kernel(basis='pivot')
K
```

Form the least-degree polynomials easily. Shortest basis vector as coefficients.

```
m = K.basis()[0]
m
```

```
p1 = sum([m[i]*x^i for i in range(5)])
p1
```

Then form the least common multiple of all five polynomials.

```
mp = lcm([p1,p2, p3, p4, p5])
mp
```

Sage can do that.

```
A.minpoly()
```

And also rational canonical form.

```
A.rational_form()
```

A precursor of rational canonical form is Zig-Zag Form.

```
B = matrix(QQ, [
[32, -74, -88, -8, 18, 58, 8, 1, -27, 23, 44, -24, 8, 1],
[-26, 14, 33, 14, 8, -32, -16, 9, -2, -15, -11, 0, 12, 0],
[-1, 23, 30, 6, 7, -21, -5, 0, -2, -9, -15, 6, 0, 10],
[-82, 101, 131, 29, -25, -95, -29, -3, 54, -30, -64, 36, -6,
-20],
[34, -36, -10, 31, 96, -20, -30, 21, -78, -13, 6, -25, 39,
59],
[-47, 44, 73, 27, 5, -60, -27, 3, 14, -22, -31, 11, 9, -5],
[-49, 133, 171, 30, 2, -123, -28, 6, 24, -48, -85, 39, -3,
22],
[39, -77, -62, 20, 80, 20, -20, 17, -74, 3, 35, -37, 37, 39],
[5, -14, -13, 9, 29, 0, -9, 4, -16, 0, 1, -5, 12, 13],
[-1, 54, 41, -24, -50, -8, 23, -9, 33, -6, -21, 22, -27,
-10],
[6, -25, -22, 5, 23, 7, -6, 8, -20, 0, 11, -11, 14, 9],
[32, -83, -107, -19, -7, 80, 19, -6, -11, 33, 55, -24, -2,
-17],
[-37, 51, 31, -31, -100, 6, 30, -16, 78, 3, -14, 26, -37,
-56],
[-54, 95, 86, -13, -77, -39, 12, -17, 80, -11, -48, 43, -34,
-39]
])
```

```
B.zigzag_form()
```