Class—SCLA SVD, Numerical Rank

Advanced Linear Algebra

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Math 390, Spring 2021

1 Rank: Exact versus Inexact/Numerical

A largish random matrix, but with lots of linear dependence.

```
n = 40
```

```
A = random_matrix(QQ, n, algorithm='echelonizable', rank=20)
print(A.str())
```

Rank. Exactly.

```
A.rank()
```

Row-reduced.

```
print(A.rref().str())
```

Now, go inexact, use floating-point approximations for all computations.

```
B = A.change_ring(RDF)
```

Rank. Wrong. Embarassing.

```
B.rank()
```

Row-reduced. Wrong too.

```
print B.rref().str()
```

What to do?

2 SVD to the Rescue

Look at "nonzero" singular values of the matrix.

```
U, S, V = B.SVD()
```

```
[S[i,i] for i in range(n)]
```

Are 20 zero and 20 nonzero?