Class—FCLA OD, Schur Decomposition

Advanced Linear Algebra

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A random 10×10 matrix.

```
A = random_matrix(ZZ, 10, 10, x=-9, y=9) + I*
    random_matrix(ZZ, 10, 10, x=-9, y=9)
A = A.change_ring(CDF)
A
```

Check the base ring, avoid the `Symbolic Ring`.

```
A.base_ring()
```

Add A to its adjoint to form B. Hermitian (self-adjoint). Normal. So a Schur decomposition is possible.

$$B^* = (A + A^*)^* = A^* + (A^*)^* = A^* + A = A + A^* = B$$

```
B = A + A.conjugate_transpose()
B
```

Compute eigenvalues and eigenvectors.

```
D, S = B.eigenmatrix_right()
```

Eigenvalues are real (a property of Hermitian matrices).

```
D.dense_matrix().round(3)
```

The eigenvectors are an orthonormal set.

```
(S.conjugate_transpose()*S).round(10)
```

The diagonalization of B.

```
(S.conjugate_transpose()*B*S).round(3)
```

As a decomposition of B.

```
(S*D*S.conjugate_transpose()).round(3)
```