Math 181 Thursday, January 28 5.7, 5.8

Fri – 6.1

Mon – 6.2

Substitution Method

Ex $\int x \sin(x^2)\,dx$

$= \frac{1}{2}(-1)\cos(x^2) + C$

$= -\frac{1}{2}\cos(x^2) + C$

check $\frac{d}{dx}\, -\frac{1}{2}\cos(x^2) + C$

$= -\frac{1}{2}\left(-\sin(x^2)(2x)\right)$

chain rule on composition

$= x\sin(x^2)$ ✓

Ex $\int \underline{x}\, \sin(\underline{\underline{x^2}})\, \underline{dx}$

$\underline{\underline{u}} = x^2 \quad du = 2x\,dx \quad \left(\frac{du}{dx} = 2x\right)$

$\frac{1}{2}du = \underline{x\,dx}$

$= \int \frac{1}{2}\sin(u)\,du = \frac{1}{2}\int \sin(u)\,dx = \frac{1}{2}\left(-\cos(u)\right) + C$

$= -\frac{1}{2}\cos(x^2) + C$

$$\int f'(g(x))\,\underbrace{g'(x)}\,dx = f(g(x)) + C$$

aftermath of the chain rule

Ex $\displaystyle\int x^3(5x^4-10)^7\,dx$ $\quad u = 5x^4-10 \quad du = 20x^3\,dx$

$\frac{1}{20}\,du = x^3\,dx$

$$= \int \frac{1}{20} u^7\,du = \frac{1}{20}\,\frac{u^8}{8} + C$$

$$= \frac{1}{160}(5x^4-10)^8 + C$$

Ex $\int_{x=1}^{x=2} x(3x^2+1)^3\,dx$

$u = 3x^2 + 1 \qquad du = 6x\,dx$

$\frac{1}{6}\,du = x\,dx$

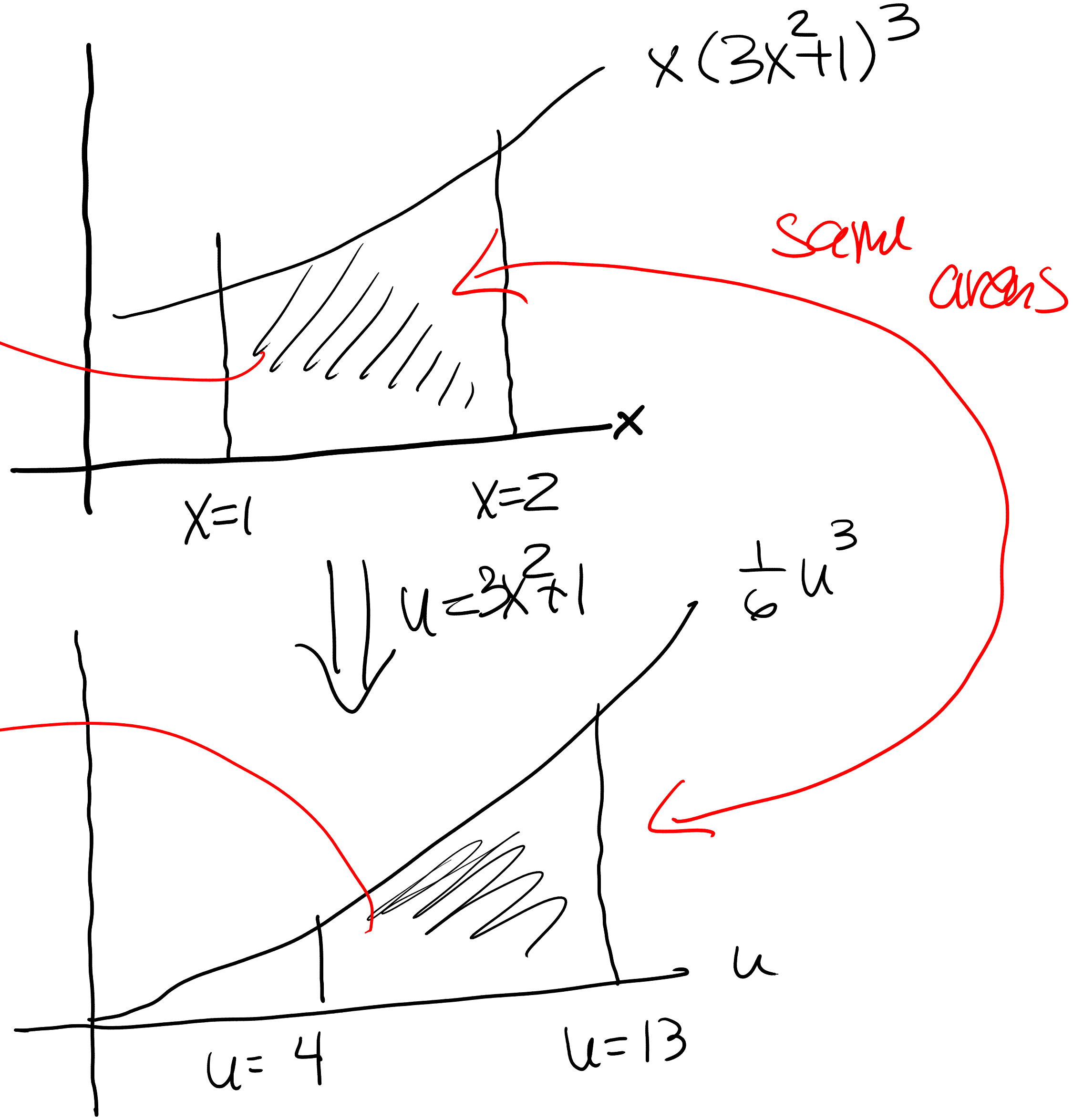
$x=1 \qquad u = 3(1)^2+1 = 4$

$x=2 \qquad u = 3(2)^2+1 = 13$

$= \int_{u=4}^{u=13} \frac{1}{6} u^3 \, du$

$= \frac{1}{24} u^4 \Big|_{u=4}^{u=13}$

$= \frac{1}{24}(13^4 - 4^4)$



Ex $\int \sin(x)\cos(x)\,dx$

$u = \sin(x) \quad du = \cos(x)\,dx$

$$\int u\,du = \frac{1}{2}u^2 + C$$

$$= \frac{1}{2}\sin^2(x) + C_1$$

Ex $\int \sin(x)\cos(x)\,dx$

$u = \cos(x) \quad du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$

$$= \int -u\,du = -\frac{1}{2}u^2 + C$$

$$= -\frac{1}{2}\cos^2(x) + C_2$$

I think constant

$$= \left(\frac{1}{2}\sin^2(x) + C_1\right) - \left(-\frac{1}{2}\cos^2(x) + C_2\right)$$

$$= \frac{1}{2}\sin^2(x) + \frac{1}{2}\cos^2(x) + C_1 - C_2$$

$$= \frac{1}{2}\left(\sin^2(x) + \cos^2(x)\right) + C_1 - C_2$$

$$= \frac{1}{2}(1) + C_1 - C_2$$

5.8 more antiderivatives

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\Longleftrightarrow$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$$