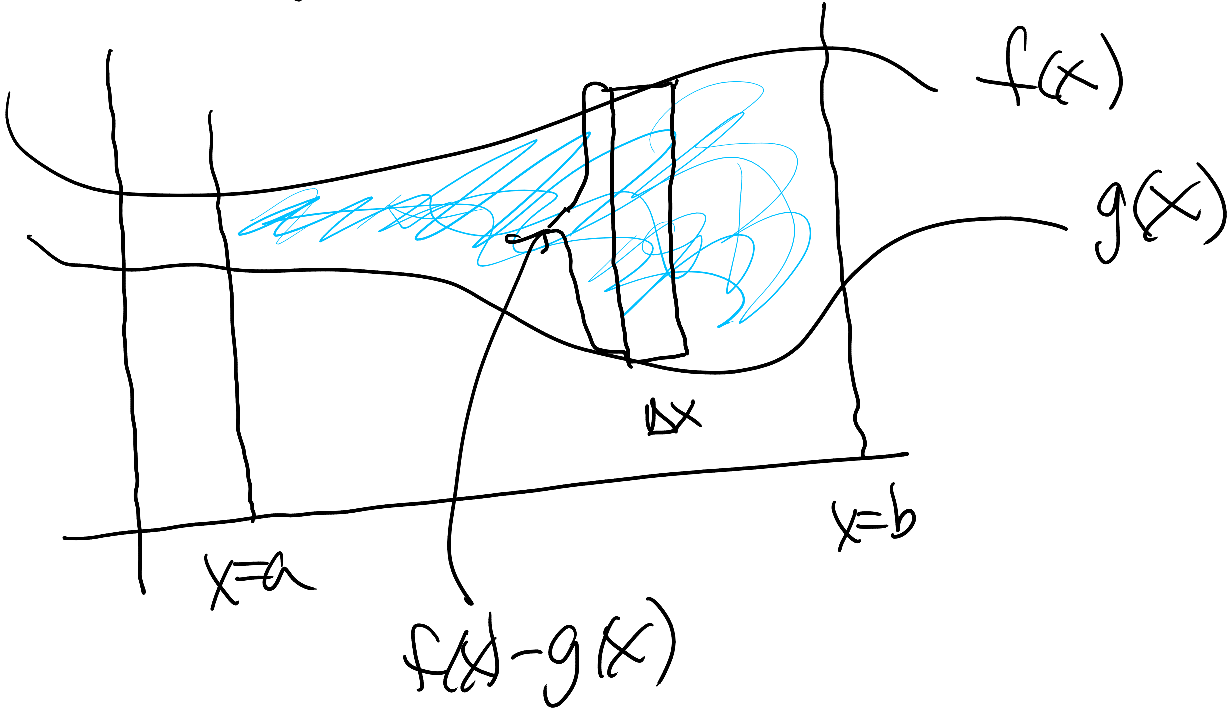
Math 181 Friday, January 29 Section 6.1

Mon - 6.2
Tue - 6.3
Thu - 6.3/6.4

Area between Curves

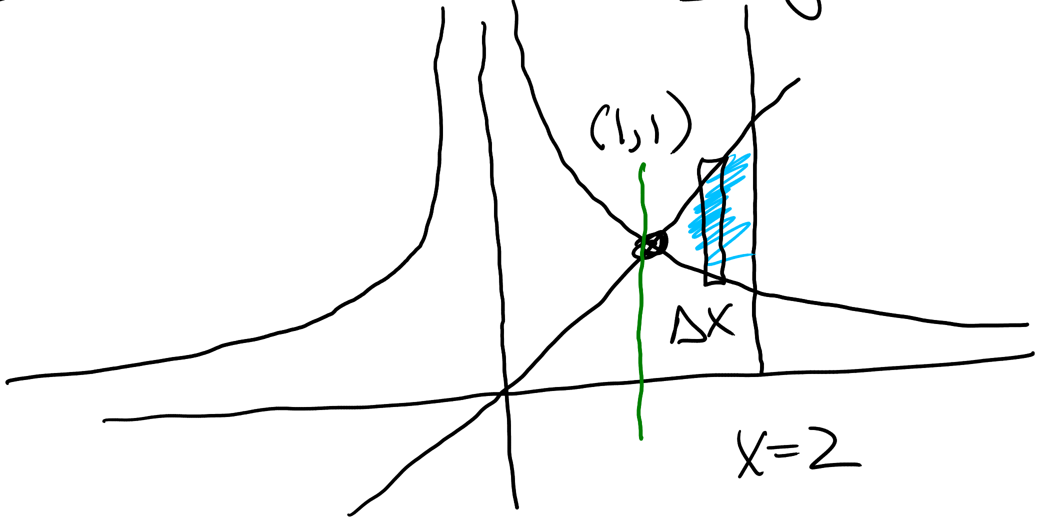
Problem: Area between $y=f(x)$ and $y=g(x)$ with $g(x) \le f(x)$ between $x=a$ & $x=b$



Area $\sum_i \left(f(x_i^*) - g(x_i^*)\right) \Delta x_i$

$$\int_{x=a}^{x=b} \left(f(x) - g(x)\right) dx$$

Ex Area bounded by $y=x$, $y=1/x^2$, & vertical line $x=2$.



$$\int_{x=1}^{x=2} x - \frac{1}{x^2} \, dx$$

one

two

third

$$\int_{x=1}^{x=2} x - x^{-2} \, dx$$

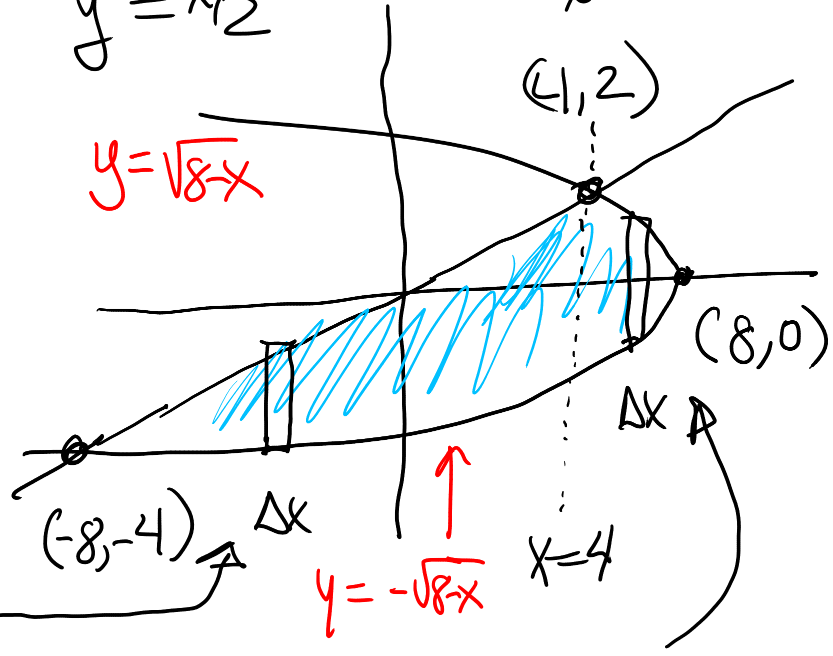
$y = 8 - x^2$

Ex Area bounded by $y = x/2$

$y^2 = 8 - x$.

$x = 8 - y^2$

$y = \pm\sqrt{8-x}$



$$\text{area} = \int_{x=-8}^{x=4} \frac{x}{2} - (-\sqrt{8x}) \, dx + \int_{x=4}^{x=8} \sqrt{8x} - (-\sqrt{8x}) \, dx$$

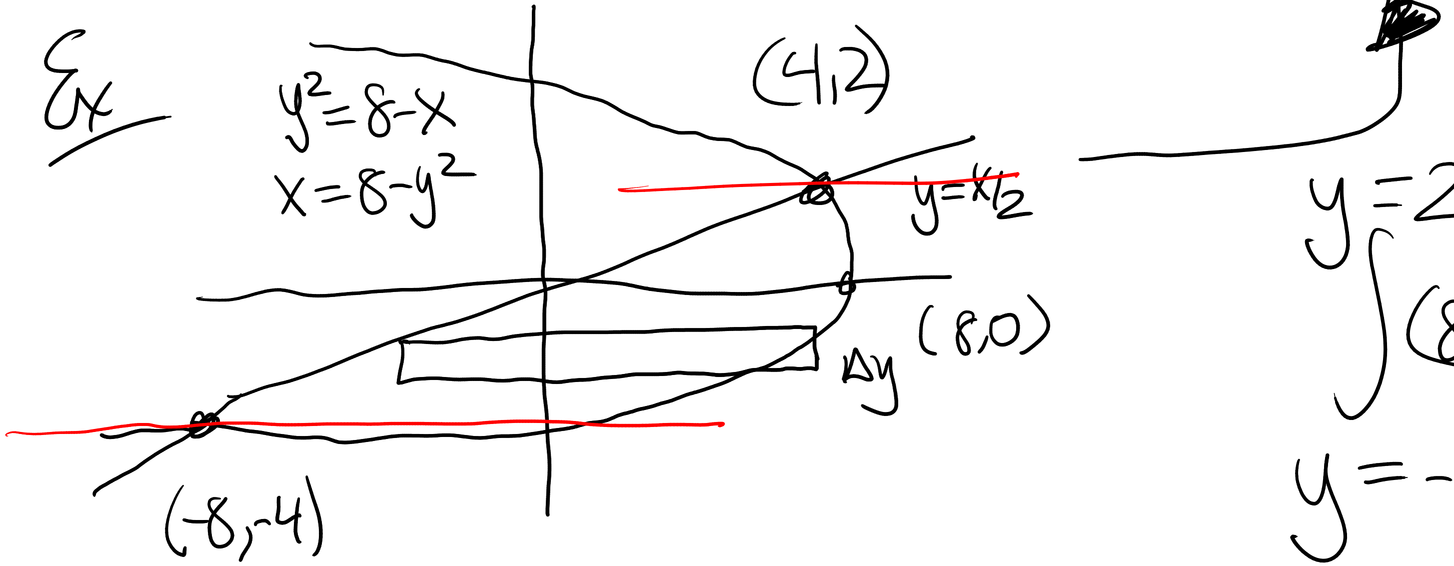
Problem: Area between $x = h(y)$ & $x = \ell(y)$ between $y = c$ & $y = d$, $h(y) \le \ell(y)$

$$\int_{y=c}^{y=d} (\ell(y) - h(y))\, dy$$

Ex

$y^2 = 8 - x$

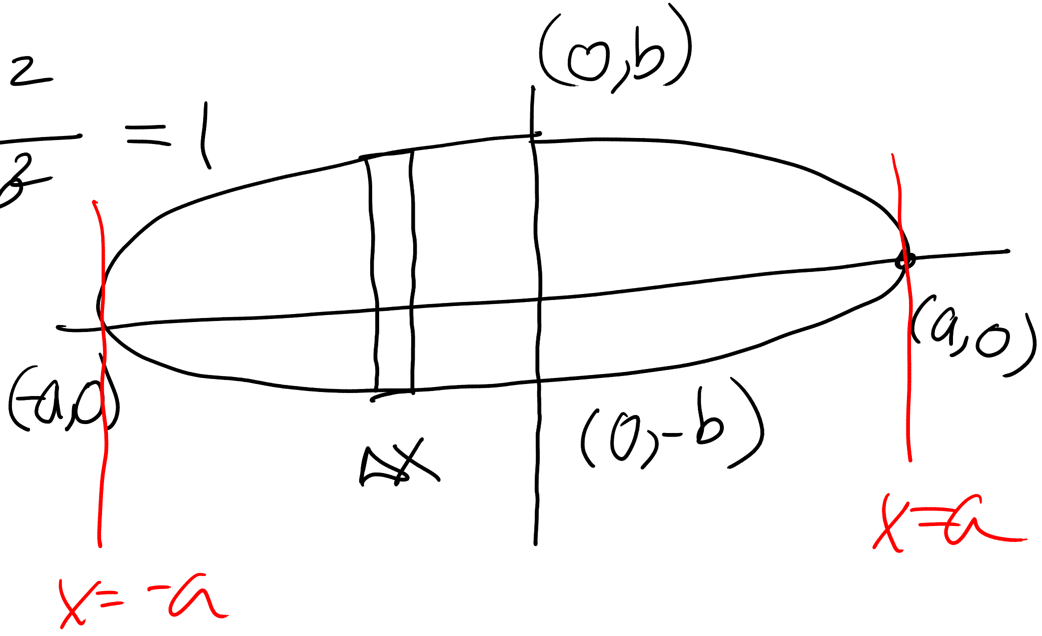
$x = 8 - y^2$



$$\int_{y=-4}^{y=2} (8-y^2) - 2y \; dy$$

$y = x/2 \rightarrow x = 2y$

Ex Area inside ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$



$$\frac{y^2}{b^2} = 1 - x^2/a^2$$

$$y^2 = b^2(1 - x^2/a^2)$$

$$y = \pm b\sqrt{1 - x^2/a^2}$$

$$\int_{x=-a}^{x=a} b\sqrt{1-x^2/a^2} - \left(-b\sqrt{1-\frac{x^2}{a^2}}\right)dx = 2b\int_{x=-a}^{x=a}\sqrt{1-x^2/a^2}\,dx$$

$$u = x/a \qquad \overset{x=au}{\wedge} \qquad du = \frac{1}{a}\,dx \rightarrow a\,du = dx$$

$$x=a \rightarrow u = a/a = 1$$

$$x=-a \rightarrow u = -a/a = -1$$

$$= 2b\int_{u=-1}^{u=1}\sqrt{1-\frac{(au)^2}{a^2}}\;a\,du$$

Area of semi-circle

$$= 2ab\int_{u=-1}^{u=1}\sqrt{1-u^2}\,du = 2ab\left(\frac{\pi(1)^2}{2}\right) = \pi ab$$