Math 181 Thursday, February 11

Section 7.1 (part 2)

Friday – 7.2 (Part 1)

Mon – Review

Tue – Exam 1
Chapters 5, 6

BREAK

Parts $\int u\,dv$?

$u = \sim\sim\sim$ $\quad dv = \sim\sim\ dx$

$du = \sim\sim\ dx$ $\quad v = \sim\sim$

$= uv - \int v\,du$

Ex $\int \ln(x)\,dx$

$u = \ln(x)$ $\quad dv = dx$

$du = \frac{1}{x}\,dx$ $\quad v = x$

$= x\ln(x) - \int x\,\frac{1}{x}\,dx$

$= x\ln(x) - \int 1\,dx = x\ln(x) - x + C$

Check: $\frac{d}{dx}\left(x\ln(x) - x\right) = \underbrace{x\left(\frac{1}{x}\right) + \ln(x)\,(1)}_{\text{Product rule!}} - 1$

$= \ln(x)$ ✓

Ex $\int \sin^{-1}(x)\, dx$

$u = \sin^{-1}(x)$ $\qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\, dx$ $\qquad v = x$

$= x\sin^{-1}(x) - \int \frac{x\, dx}{\sqrt{1-x^2}}$

$u = 1-x^2$
substitution

Ex $\int_{x=0}^{x=\pi/2} x\cos(x)\, dx$

$u = x$ $\qquad dv = \cos(x)\, dx$

$du = dx$ $\qquad v = \sin(x)$

$= x\sin(x)\Big|_{x=0}^{x=\pi/2} - \int_{x=0}^{x=\pi/2} \sin(x)\, dx$

$= x\sin(x)\Big|_{x=0}^{x=\pi/2} + \cos(x)\Big|_{x=0}^{x=\pi/2}$

$= \left[\pi/2 \sin(\pi/2) + \cos(\pi/2)\right] - \left[0\sin(0) + \cos(0)\right]$

$=$ number

Ex $\int e^{5x}\cos(4x)\,dx = I$

$u = \cos(4x) \qquad dv = e^{5x}\,dx$

$du = -4\sin(4x)\,dx \qquad v = \frac{1}{5}e^{5x}$

$= \frac{1}{5}e^{5x}\cos(4x) - \left(\frac{-4}{5}\right)\int e^{5x}\sin(4x)\,dx$

$u = \sin(4x) \qquad dv = e^{5x}\,dx$

$du = 4\cos(4x)\,dx \qquad v = \frac{1}{5}e^{5x}$

$I = \frac{1}{5}e^{5x}\cos(4x) + \frac{4}{5}\left[\frac{1}{5}e^{5x}\sin(4x) - \left(\frac{4}{5}\right)\int e^{5x}\cos(4x)\,dx\right]$

$= \frac{1}{5}e^{5x}\cos(4x) + \frac{4}{25}e^{5x}\sin(4x) - \frac{16}{25}I$

$41/25$

$\left(1+\frac{16}{25}\right)I = \frac{1}{5}e^{5x}\cos(4x) + \frac{4}{25}e^{5x}\sin(4x)$

$I = \frac{25}{41}\left(\frac{1}{5}e^{5x}\cos(4x) + \frac{4}{25}e^{5x}\sin(4x)\right) + C$

$= \frac{5}{41}e^{5x}\cos(4x) + \frac{4}{41}e^{5x}\sin(4x) + C$

Ex $\int x^3\sqrt{1-x^2}\,dx$

$u = x^2$ $\quad dv = x\sqrt{1-x^2}\,dx$

$du = 2x\,dx$ $\quad v = -\frac{1}{3}(1-x^2)^{3/2}$

$\frac{d}{dx}v = \frac{d}{dx}\ -\frac{1}{3}(1-x^2)^{3/2}$

$= -\frac{1}{3}\left(\frac{3}{2}\right)(1-x^2)^{1/2}(-2x)$

$t = 1-x^2,\ dt = -2x\,dx$

$-\frac{1}{2}dt = x\,dx$

t- substitution

$= x^2\left(-\frac{1}{3}(1-x^2)^{3/2}\right) - \int -\frac{2}{3}x(1-x^2)^{3/2}\,dx$

$t = 1-x^2,\ dt = -2x\,dx$

$-\frac{1}{2}dt = x\,dx$

PARTS

Ex $\int (\ln(x))^n\,dx = x(\ln(x))^n - n\int(\ln(x))^{n-1}\,dx$

"reduction formula"