Math 181 Friday, February 12 Section 7.2 (part 1)

WW 7.1 Good

WW 7.2-preview
due 2021/02/22

Mon - Review / Problems

Tue - Exam
Chapter 5, 6

BREAK

7.2 Trigonometric Integrals

Ex $\int \cos^5(x) \sin^6(x)\, dx$

$= \int \cos(x) \cos^4(x) \sin^6(x)\, dx$

$= \int \cos(x) \left(\cos^2(x)\right)^2 \sin^6(x)\, dx$

$= \int \cos(x) \left(1-\sin^2(x)\right)^2 \sin^6(x)\, dx$

$u = \sin(x) \qquad du = \cos(x)\, dx$

$= \int (1-u^2)^2 u^6 \; du$

$= \int (1-2u^2+u^4)\, u^6\, du$

$= \int u^6 - 2u^8 + u^{10}\, du$

$= \frac{1}{7}u^7 - \frac{2}{9}u^9 + \frac{1}{11}u^{11} + C$

$= \frac{1}{7}\sin^7(x) - \frac{2}{9}\sin^9(x) + \frac{1}{11}\sin^{11}(x) + C$

Odd power of cosine + even power of sine (or vice versa)

Strip out one odd power for differential, convert remaining even power w/ $\sin^2(x)+\cos^2(x)=1$, then u-substitution

Even power of cosine & even power of sine

Repeated use $\sin^2(x)=\frac{1}{2}(1-\cos(2x))$, $\cos^2(x)=\frac{1}{2}(1+\cos(2x))$

ex

$$\int \cos^2(x)\sin^4(x)\,dx = \int \frac{1}{2}(1+\cos(2x))(\sin^2(x))^2\,dx$$

$$= \int \frac{1}{2}(1+\cos(2x))\left(\frac{1}{2}(1-\cos(2x))\right)^2 dx$$

$$= \frac{1}{8}\int (1+\cos(2x))(1-2\cos(2x)+\cos^2(2x))\,dx$$

$$= \frac{1}{8}\int 1-2\cos(2x)+\cos^2(2x)+\cos(2x)-2\cos^2(2x)+\cos^3(2x)\,dx$$

$$= \frac{1}{8}\int 1+\cos(2x)-\cos^2(2x)+\cos^3(2x)\,dx$$

EZ EZ

$\cos^2(2x)=\frac{1}{2}(1+\cos(4x))$

odd power of cosine

Any power of tan(x) w/ even power of sec(x)

$$\int \tan^n(x) \sec^m(x)$$ ← even

Ex $\int \tan^8(x) \sec^6(x)\,dx$

$= \int \tan^8(x) \sec^4(x) \sec^2(x)\,dx$ Pull out $\sec^2(x)$ for differential

$= \int \tan^8(x) \left(\sec^2(x)\right)^2 \sec^2(x)\,dx$

$= \int \tan^8(x) \left(1+\tan^2(x)\right)^2 \sec^2(x)\,dx$

$1+\tan^2(x) = \sec^2(x)$

$u = \tan(x) \quad du = \sec^2(x)\,dx$

$= \int u^8 (1+u^2)^2\,du$

$= \int u^8(1+2u^2+u^4)\,du = \int u^8+2u^{10}+u^{12}\,du$

$= \frac{1}{9}u^9 + \frac{2}{11}u^{11} + \frac{1}{13}u^{13} + C = \frac{1}{9}\tan^9(x) + \frac{2}{11}\tan^{11}(x) + \frac{1}{13}\tan^{13}(x) + C$

Odd power of $\tan(x)$

$$\int \tan^5(x)\sec^3(x)\,dx$$

$$= \int \tan^4(x)\sec^2(x)\,(\tan(x)\sec(x))\,dx$$

set up differential

$$= \int (\tan^2(x))^2 \sec^2(x)\tan(x)\sec(x)\,dx$$

$$= \int (\sec^2(x)-1)^2 \sec^2(x)\tan(x)\sec(x)\,dx$$

$1+\tan^2(x) = \sec^2(x)$

$u = \sec(x) \quad du = \sec(x)\tan(x)\,dx$

$$= \int (u^2-1)^2 u^2\,du$$

$$= \int (u^4-2u^2+1)u^2\,du = \int u^6-2u^4+u^2\,du$$

$$= \frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3 + C = \frac{1}{7}\sec^7(x) - \frac{2}{5}\sec^5(x) + \frac{1}{3}\sec^3(x) + C$$