Math 181 Thursday, February 25 Section 7.3 (part 2)

Fri 7.4
BYOB: movies

Mon 7.5

Tue 7.5/7.7

Thu 7.7

Integrand	Substitution	Yields
$\sqrt{a^2-x^2}$	$x=a\sin\theta$	$a\cos\theta$
$\sqrt{x^2-a^2}$	$x=a\sec\theta$	$a\tan\theta$
$\sqrt{x^2+a^2}$	$x=a\tan\theta$	$a\sec\theta$

$(\ )^{3/2}, (\ )^{-7/2}, (\ )^{1}$

Ex $\displaystyle\int \frac{dx}{x\sqrt{4-x^2}}$ $\quad x = 2\sin\theta$, $\quad dx = 2\cos\theta\, d\theta$

$\ln(a/b) = \ln a - \ln b$

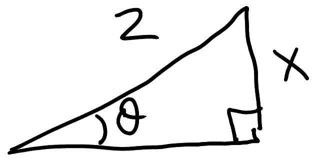
$$= \int \frac{2\cos\theta\, d\theta}{2\sin\theta\sqrt{4-(2\sin\theta)^2}} = \int \frac{\cos\theta\, d\theta}{\sin\theta\left(2\sqrt{1-\sin^2\theta}\right)}$$

$$= \frac{1}{2}\int \frac{\cos\theta\, d\theta}{\sin\theta\sqrt{\cos^2\theta}} = \frac{1}{2}\int \frac{\cos\theta\, d\theta}{\sin\theta\cos\theta} = \frac{1}{2}\int \csc\theta\, d\theta$$

$$= -\frac{1}{2}\ln|\csc\theta + \cot\theta| + C \rightarrow -\frac{1}{2}\ln\left|\frac{2}{x} + \frac{\sqrt{4-x^2}}{x}\right| + C = -\frac{1}{2}\ln\left|\frac{2+\sqrt{4-x^2}}{x}\right| + C$$

$x = 2\sin\theta$

$\frac{x}{2} = \sin\theta \Rightarrow$



so $\sqrt{4-x^2}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{(x/2)} = 2/x$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{x}{\sqrt{4-x^2}}\right)} = \frac{\sqrt{4-x^2}}{x}$

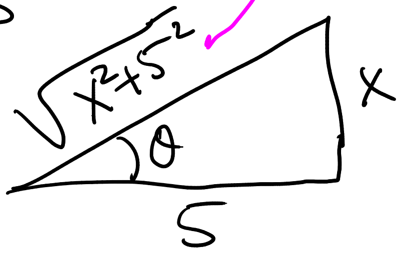
Ex $\displaystyle\int \frac{dx}{\sqrt{25+x^2}}$ $\quad x = 5\tan\theta$ $\quad dx = 5\sec^2\theta\, d\theta$ $\qquad \int$ vs. $\int_2^3$

$$= \int \frac{5\sec^2\theta\, d\theta}{\sqrt{25+(5\tan\theta)^2}} = \int \frac{5\sec^2\theta\, d\theta}{5\sqrt{1+\tan^2\theta}} = \int \frac{\sec^2\theta\, d\theta}{\sqrt{\sec^2\theta}}$$

$$= \int \sec\theta\, d\theta = \ln(|\sec\theta + \tan\theta|) + C$$

$x = 5\tan\theta$

$\frac{x}{5} = \tan\theta$



$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(5/\sqrt{x^2+25}\right)}$

$\tan\theta = x/5$

$$= \ln\left(\left|\frac{\sqrt{x^2+25}}{5} + \frac{x}{5}\right|\right) + C$$

$$= \ln\left(\left|\frac{\sqrt{x^2+25}+x}{5}\right|\right) + C$$

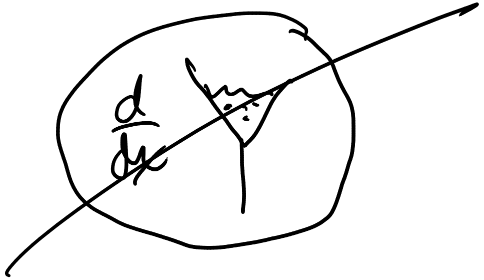
$$= \ln\left(\frac{|\sqrt{x^2+25}+x|}{5}\right) + C$$

$$= \ln|\sqrt{x^2+25}+x| - \ln 5 + C$$

$$= \ln(|\sqrt{x^2+25}+x|) + C$$

Ex

$$\int \frac{x^2\,dx}{(x^2-1)^{5/2}} \qquad x = \sec\theta \qquad dx = \sec\theta\tan\theta\,d\theta$$



$$= \int \frac{\sec^2\theta\,(\sec\theta\tan\theta)\,d\theta}{(\sec^2\theta-1)^{5/2}} = \int \frac{\sec^3\theta\tan\theta\,d\theta}{(\tan^2\theta)^{5/2}} = \int \frac{\sec^3\theta\tan\theta\,d\theta}{\tan^5\theta}$$

$$= \int \frac{1}{\cos^3\theta}\cot^4\theta\,d\theta = \int \frac{1}{\cos^3\theta}\,\frac{\cos^4\theta}{\sin^4\theta}\,d\theta = \int \frac{\cos\theta\,d\theta}{\sin^4\theta}$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

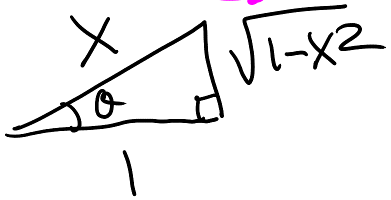
$$= \int \frac{du}{u^4} = \int u^{-4}\,du = \frac{1}{-3}u^{-3} + C = \frac{-1}{3}(\sin\theta)^{-3} + C$$

$$= \frac{-1}{3}\left(\frac{\sqrt{1-x^2}}{x}\right)^{-3} + C$$

so $\sin\theta = \dfrac{\sqrt{1-x^2}}{x}$

$\frac{x}{1} = \sec\theta$

$\frac{1}{x} = \cos\theta$



$$= \frac{-1}{3}\left(\frac{x}{\sqrt{1-x^2}}\right)^3 + C = \frac{-x^3}{3(1-x^2)^{3/2}} + C$$