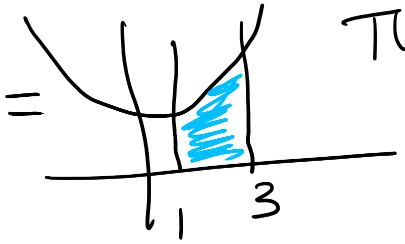
Math 181 Friday, March 5 Section 7.8 (part 1)

Mon - 7.8

Tue - 10.1

Numerical Integration

Definite Integrals $\int_1^3 x^2+4x\,dx =$ 

= number

= measurement

Indefinite Integral (Antiderivative)

$\int x^2+4x\,dx$ = function

Compute a definite integral easily w/ an antiderivative.

Fundamental Theorem of Calculus.

What about $\int_1^5 e^{-x^2}\,dx$? e^{-x^2} does not have a "nice" antiderivative!

Can't use FTC. (There $\underline{\text{is}}$ an antiderivative : $g(x) = \int_0^x e^{-t^2}\,dt$.)

So we want/need to approximate definite integrals.

Why antidifferentiate?

<u>Physics</u>

$$a(t) = g$$

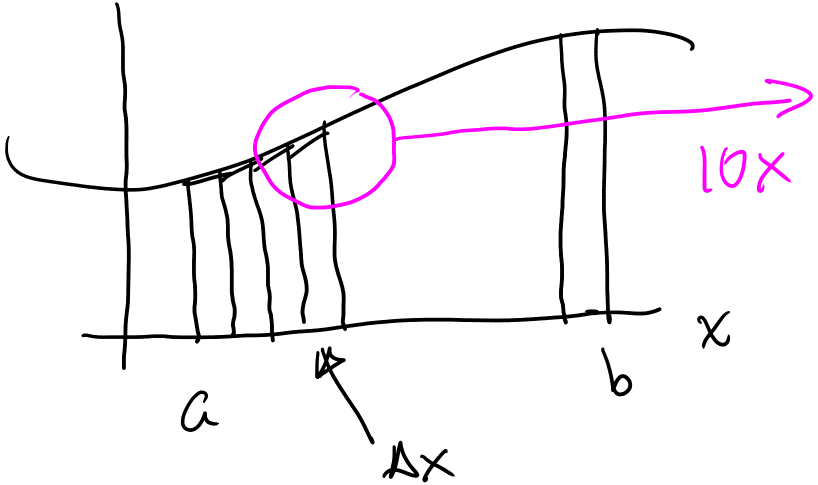
$$v(t) = \int g\,dt = gt + C \longrightarrow v(t) = gt + v_0$$

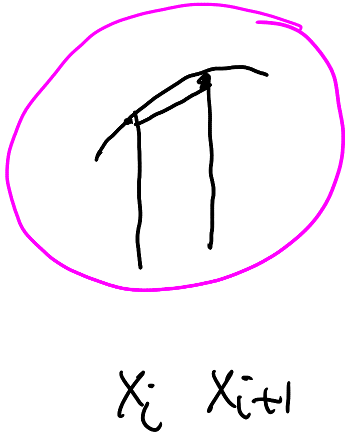
$$s(t) = \int v(t)\,dt = \int gt + v_0\,dt = \frac{1}{2}gt^2 + v_0 t + C$$

$$\longrightarrow s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$$

Trapezoidal Rule: use "rectangles" with slanted tops

$\int_a^b f(x)\,dx$





$$\text{Area} = \left(\frac{f(x_i) + f(x_{i+1})}{2}\right)\Delta x$$

Sum of these areas

$$T_n = \left(\tfrac{1}{2} f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \tfrac{1}{2} f(x_n)\right)\Delta x$$

Ex $\int_1^3 x^3\,dx$ $(= 20$ by FTC$)$

$n = 8$

$\Delta x = \frac{3-1}{8} = 1/4$

$x_0 = 1$ $\quad x_4 = 2$

$x_1 = 1\,1/4$ $\quad x_5 = 2\,1/4$

$x_2 = 1\,1/2$ $\quad x_6 = 2\,1/2$

$x_3 = 1\,3/4$ $\quad x_7 = 2\,3/4$

$x_8 = 3$

$$T_8 = \left(\tfrac{1}{2} f(x_0) + f(x_1) + \cdots + f(x_7) + \tfrac{1}{2} f(x_8)\right)\Delta x$$

$$= \left(\tfrac{1}{2}(1^3) + (1\,1/4)^3 + \cdots + (2\,3/4)^3 + \tfrac{1}{2}(3)^3\right)\tfrac{1}{4}$$

$$= \frac{161}{8} = 20\,1/8$$

$n=100 \qquad T_{100} = \dfrac{25\,001}{1250} = 20\,\dfrac{1}{1250}$

$\underline{Ex}$ $\displaystyle\int_1^5 e^{-x^2}\,dx$

$T_8 = \left(\frac{1}{2}e^{-1^2} + e^{-(1\frac{1}{2})^2} + e^{-(2)^2} + \cdots + e^{-(4\frac{1}{2})^2} + \frac{1}{2}e^{-5^2}\right)\frac{1}{2}$

$n=8, \ \Delta x = \dfrac{5-1}{8} = \dfrac{1}{2}$

$f(x) = e^{-x^2}$

$= 0.15485$

$T_{100} = 0.139500899$

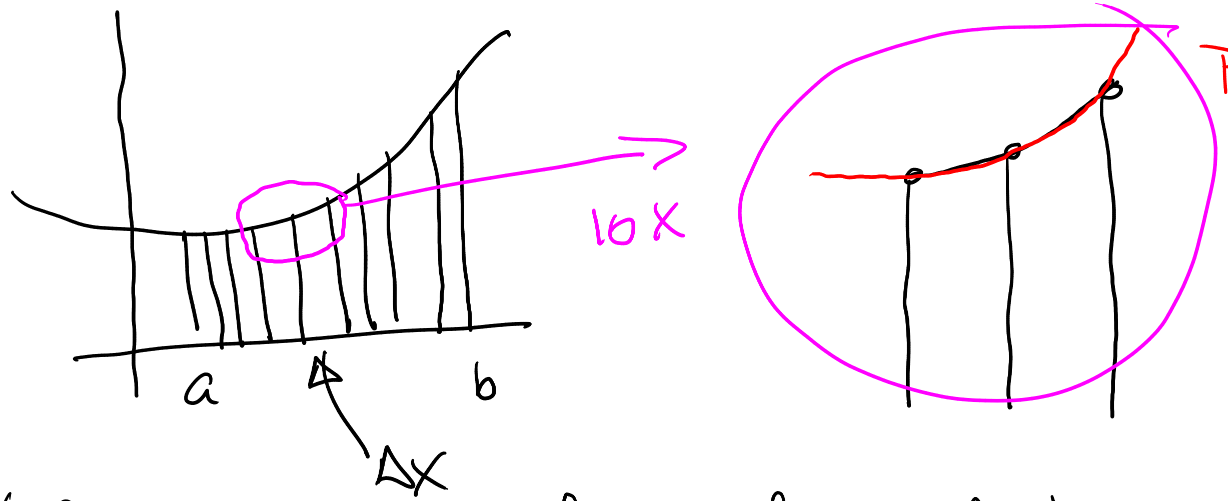
8 rectangles, maximum on each rectangle 0.24682 (overestimate)

8 rectangles, minimum on each rectangle 0.0662 (underestimate)

Simpson's Rule: Same idea, but <u>quadratic</u> tops.

<u>Even</u> number of points on x-axis

$\int_a^b f(x)\,dx$



Parabola through <u>three</u> points.

$$S_n = \frac{1}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 4f(x_{n-1}) + f(x_n) \right) \Delta x$$

Ex $\int_1^3 x^4\,dx$ $\left(\text{FTC}: = \frac{242}{5} = 48.4 \right)$

$n = 8$ $S_8 = \frac{9293}{192} = 48.401041$