Math 181 Friday, March 12 Section 10.2

Mon - Review (7.8 ww)

Tue - Exam 2
Chapter 7

Thu - 10.3

Fri - 10.3/10.4

Ex Pierce County build a compost facility.
Expect 100 tons of yard waste each week
Extract 30% of material each week for use.
How big should the facility be?

Week 1: 100

Week 2: $100 + \underbrace{100(.70)}_{\text{last week, less 30\%}}$

Week 3: $100 + \underbrace{(100 + 100(.70))(.70)}_{\text{70\% of current}}$

$100 + 100(.70) + 100(.70)^2$

Week 4: $100 + (100 + 100(.70) + 100(.70)^2)(.70)$

$= 100 + 100(.70) + 100(.70)^2 + 100(.70)^3$

$\vdots$

Week n $= 100 + 100(.70) + 100(.70)^2 + \cdots + 100(.70)^{n-1}$

$\vdots$

Week ∞ $= 100 + 100(.70) + 100(.70)^2 + \cdots$

Infinite series $\displaystyle\sum_{i=0}^{\infty} 100(.70)^i$

Defn Infinite Series.

Base sequence a_n. New sequence of partial sums.

$$S_n = a_1 + a_2 + \cdots + a_n$$

Then the infinite series is

$$\sum_{i=1}^{\infty} a_i = \lim_{n\to\infty} S_n$$

Geometric Series

$$\sum_{i=0}^{\infty} \underbrace{ar^i}_{\text{base sequence } a_i = ar^i} = \frac{a}{1-r} \quad \text{if} \quad |r|<1$$

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^n$$

$$+ \quad -rS_n = \quad -ar - ar^2 - ar^3 - \dots - ar^n - ar^{n+1}$$

$$S_n + (-r)S_n = a + (-ar^{n+1})$$

$$(1-r)\,S_n = a(1-r^{n+1})$$

$$S_n = \frac{a(1-r^{n+1})}{1-r}$$

only if $|r|<1$

$$\sum_{i=0}^{\infty} ar^i = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a(1-r^{n+1})}{1-r} = \frac{a(1-0)}{1-r} = \frac{a}{1-r}$$

So for compost facility: $\displaystyle\sum_{i=0}^{\infty} 100(.70)^i = \frac{100}{1-.70} = \frac{100}{.3} = \frac{10}{3}100 = \frac{1000}{3}$
$= 333.3$

Ex $\displaystyle\sum_{n=2}^{\infty} \frac{3}{(n+2)(n-1)}$

$$= \underset{n=2}{3/4} + \underset{n=3}{3/10} + \underset{n=4}{3/18} + \underset{n=5}{3/28} + \cdots\cdots$$

Algebra: $\displaystyle\frac{3}{(n+2)(n-1)} = \frac{1}{n-1} + \frac{-1}{n+2}$

$$S_n = \frac{3}{(2+2)(2-1)} + \frac{3}{(3+2)(3-1)} + \cdots + \frac{3}{(n+2)(n-1)}$$ telescoping

$$= \left(\frac{1}{2-1} + \cancel{\frac{-1}{2+2}}\right) + \left(\frac{1}{3-1} + \cancel{\frac{-1}{3+2}}\right) + \left(\frac{1}{4-1} + \frac{-1}{4+2}\right) + \left(\cancel{\frac{1}{5-1}} + \frac{-1}{5+2}\right) + \cancel{\cdots} + \left(\frac{1}{n-1} + \frac{-1}{n+2}\right)$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{-1}{n} + \frac{-1}{n+1} + \frac{-1}{n+2}$$

$$\lim_{n\to\infty} S_n = 1\tfrac{5}{6} + 0 + 0 + 0$$