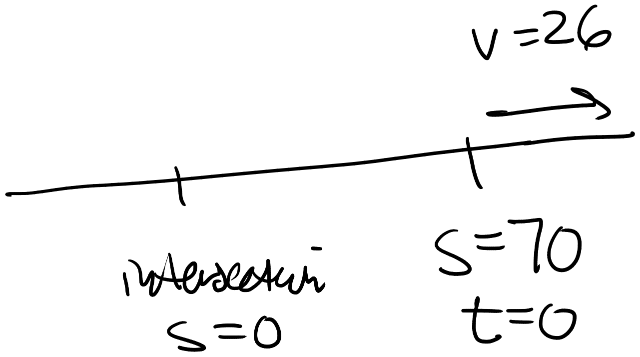
Math 181 — Monday, April 5 — Section 10.8

WW 10.7.5



$s(t)$ – position

$s(0) = 70$

$s' = v(0) = 26$

$s'' = a(0) = -5$

$$T_2(t) = 70 + 26(t-0) + \frac{-5}{2!}(t-0)^2$$

$$= 70 + 26t - \tfrac{5}{2}t^2$$

$$s(8) \cong T_2(8) = 70 + 26(8) - \tfrac{5}{2}(8)^2$$

Tue 8.1

Thu 8.1 / 8.2

Fri 8.2

Mon Review

Tue Exam 3
Chapter 10

No 8.1 - preview

Final Exam
Wed, May 12, Noon
→ 3 PM

Ex Find a Taylor polynomial for $m(x) = 2x^3 - 5x^2 + 2x - 3$ about $a = 1$.

$m(x) = 2x^3 - 5x^2 + 2x - 3$ $\qquad m(1) = -4$

$m'(x) = 6x^2 - 10x + 2$ $\qquad m'(1) = -2$

$m''(x) = 12x - 10$ $\qquad m''(1) = 2$

$m^{(3)}(x) = 12$ $\qquad m^{(3)}(1) = 12$

$m^{(4)}(x) = 0$ $\qquad m^{(4)}(1) = 0$

$m^{(5)}(x) = 0$ $\qquad m^{(5)}(1) = 0$

$\vdots \qquad \vdots$

$$T_3(x) = -4 + -2(x-1) + \frac{2}{2!}(x-1)^2 + \frac{12}{3!}(x-1)^3$$

$$= \underline{-3 + 2x - 5x^2 + 2x^3}$$

<u>Theorem</u> $f(x)$ - function $T_n(x)$ Taylor series about $x=a$, then

$$\underbrace{|\,f(x) - T_n(x)\,|}_{\text{error}} = \frac{\boxed{f^{n+1}(z)}}{(n+1)!}(x-a)^{n+1}$$

where z is <u>between</u> x & a.

$f(x)$: function

$T_n(x)$: approximating polynomial

$\frac{f^{n+1}(z)}{(n+1)!}(x-a)^{n+1}$: looks like the next term of the polynomial

x: where used

a: where built

z: not known

<u>upper bound on $f^{n+1}(z)$</u> (How bad?)

<u>Ex</u> Estimate e with a degree 6 Taylor polynomial, $f(x)=e^x$, $a=0$

$$T_6(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!}$$

$$e = e^1 = f(1) \cong T_6(1) = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} = 2.7180\overline{5}$$

Remainder term $\frac{f^7(z)}{7!}(1-0)^7 = \frac{e^z}{7!}$ where $0 \le z \le 1$

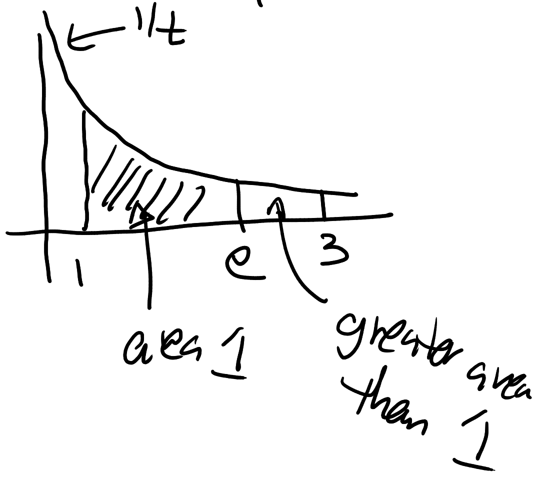
$$\frac{e^z}{7!} \le \frac{e^1}{7!} \le \frac{3}{7!}$$

$$= .0005952$$

$$\ln(x) = \int_1^x \frac{1}{t}\,dt$$

$$\ln(e) = \int_1^e 1/t\,dt$$

$$1 = \int_1^e \frac{1}{t}\,dt$$



We say $e^x \boxed{=} 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

function power series

Radius of convergence: $(-\infty, \infty)$ (Ratio Test)

Or $R_n(x) = \frac{f^{n+1}(z)}{(n+1)!}(x-a)^{n+1} = \frac{e^z (x-a)^{n+1}}{(n+1)!} \le \frac{3^z (x-a)^{n+1}}{(n+1)!}$

$\lim_{n\to\infty}$

$= 0$

Power Series solutions to Differential Equations

Differential Equation: $\frac{dy}{dx} = xy$

Solution

$$y = \sum_{k=1}^{\infty} \frac{x^{3k}}{(2\cdot 3)(5\cdot 6)\cdots((3k-1)(3k))}$$

$\frac{dy}{dx} = y$, $y = e^x$

$\frac{dy}{dx} = 3x^2$, $y = x^3$