

Math 181

Thursday, April 15

Differential Equations

$$\frac{dy}{dx} = 2x, \quad y? \Rightarrow y = x^2 + C$$

Initial condition: $x=3, y=12$

$$12 = y = 3^2 + C \rightarrow 3 = C$$

$$\text{So } y = x^2 + 3$$

Similar to $\int 2x \, dx$

indefinite integral or antiderivative

Section 9.1

Fri - 9.1/9.2

Mon - 9.2

BYOB FRI **FOOD**

simple differential equation

Ex $y + y'' = 0$, what is y ?

$y = \sin(x)$ is one solution

$y = \cos(x)$ is another

$y = 10 \sin(x)$

$y = \sin(x) + \cos(x)$

$y = 8 \sin(x) + 12 \cos(x)$

so

$y = a \sin(x) + b \cos(x)$

is a solution for all a & b

differential equation, solution is a function

$$x^2 - 2x + 1 = 0$$

solution to this equation is a number(s)

Math 301 | Differential Equations
Math 302

Separable Differential Equations

$$\frac{dy}{dx} = f(x)g(y)$$

$$\frac{1}{g(y)} dy = f(x) dx$$

$$\int \frac{1}{g(y)} dy = \int f(x) dx$$

Solve for y , track
constant of integration carefully

$$\underline{\text{Ex}} \quad \frac{dy}{dx} = 4yx^3, \quad y(0) = 2$$

$$\frac{1}{y} dy = 4x^3 dx$$

$$\int \frac{1}{y} dy = \int 4x^3 dx$$

$$\ln y = x^4 + C$$

$$e^{\ln y} = e^{x^4 + C}$$

$$y = e^{x^4} e^C$$

$$= K e^{x^4} \quad (K > 0)$$

$$2 = y(0) = K e^{2^4} = K e^{16}$$

$$\Rightarrow K = 2e^{-16}$$

$$y = 2e^{-16} e^{x^4}$$
$$= 2e^{x^4 - 16}$$

Ex $\frac{dy}{dx} = \frac{x^2}{y^2}, \underline{\underline{y(0) = 2}}$

$$\frac{dy}{dx} = (x^2) \left(\frac{1}{y^2} \right)$$

separable

$$y^2 dy = x^2 dx$$

$$\int y^2 dy = \int x^2 dx$$

$$\frac{y^3}{3} = \frac{x^3}{3} + C$$

$$y^3 = x^3 + C$$

$$y = (x^3 + C)^{1/3}$$

$$2 = y(0) = (0^3 + C)^{1/3}$$

$$2 = C^{1/3} \rightarrow C = 2^3 = 8$$

$$\text{So } y = (x^3 + 8)^{1/3}$$