Math 181 Monday, April 26 Section 11.1

Tue 11.1/11.2

Thu 11.2

Fri Problems

BYOB Summer

A curve is described by two equations $x(t)$ & $y(t)$, $a \le t \le b$

parameter t

Ex $x(t) = 3t+2$ $y(t) = -2t+6$ $0 \le t \le 4$

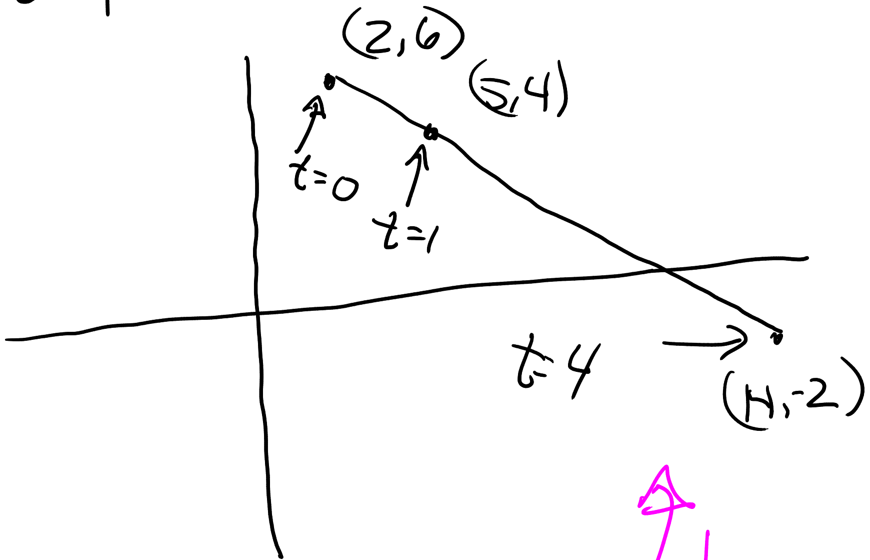
$t=0$ $(x,y) = (x(0), y(0)) = (2,6)$

$t=1$ $(x,y) = (x(1), y(1)) = (5,4)$

$t=2$ $(x,y) = (x(2), y(2)) = (8,2)$

$t=3$ $(x,y) = (x(3), y(3)) = (11,0)$

$t=4$ $(x,y) = (x(4), y(4)) = (14,-2)$



More information here

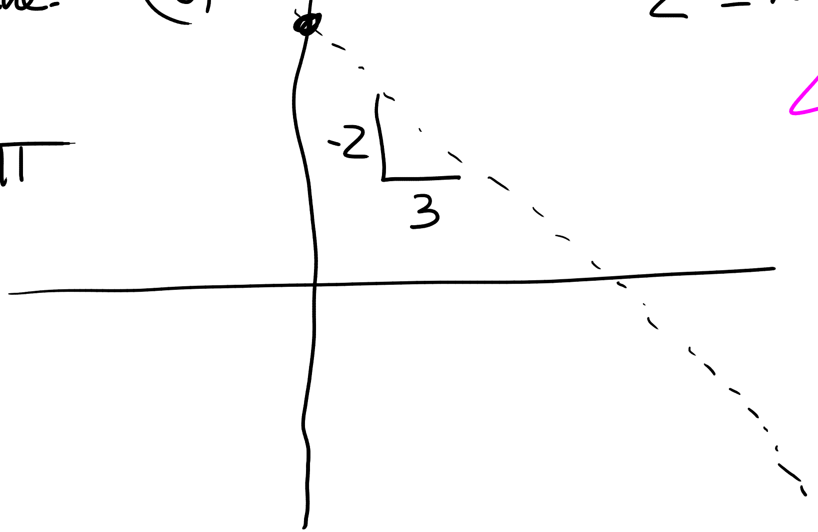
not here

Shape of this curve? Eliminate the parameter,

$x = 3t+2 \rightarrow \frac{1}{3}(x-2) = t \Rightarrow y = -2t+6 = -2\left(\frac{1}{3}(x-2)\right)+6$

$= -\frac{2}{3}x + \frac{4}{3} + 6 = -\frac{2}{3}x + \frac{22}{3}$

The points on this curve lie in a line. $(0, 22/3)$ $\quad 2 \le x \le 14$



Ex $x(t) = 4 + 2\cos(t)$ $\quad 0 \le t \le 4\pi$

$y(t) = -1 + 2\sin(t)$

Eliminate the parameter

$x - 4 = 2\cos(t)$

$y + 1 = 2\sin(t)$

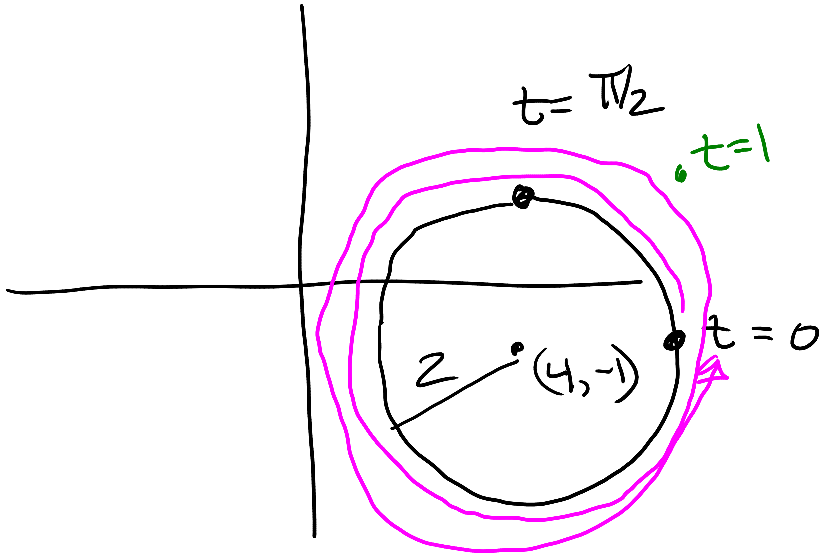
$(2\cos(t))^2 + (2\sin(t))^2 = 4\cos^2(t) + 4\sin^2(t) = 4(\cos^2(t) + \sin^2 t) = 4 \cdot 1 = 4$

equal

$= (x-4)^2 + (y+1)^2$

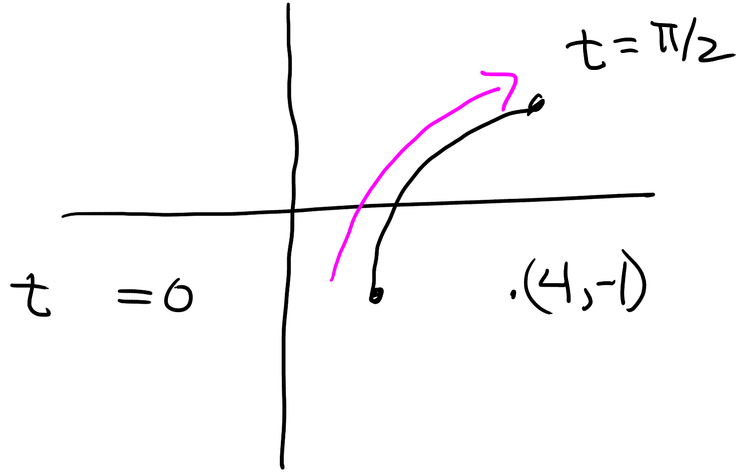
or

So $(x-4)^2 + (y+1)^2 = 4 = 2^2$ $\quad$ Circle w/ center at $(4,-1)$, radius 2



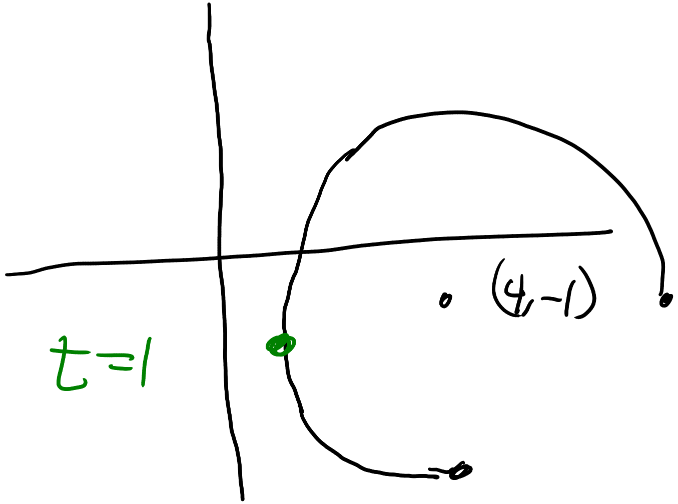
Ex $x(t) = 4 - 2\cos(t)$ $\quad 0 \le t \le \pi/2$

$y(t) = -1 + 2\sin(t)$



Ex $x(t) = 4 + 2\cos(4t)$ $\quad 0 \le t \le \frac{3\pi}{8}$

$y(t) = -1 + 2\sin(4t)$



Ex

$x(t) = 4 + 2\cos(t)$

$y(t) = -1 + 5\sin(t) \qquad 0 \le t \le \pi$

Eliminate the parameter

$\frac{1}{2}(x-4) = \cos(t)$

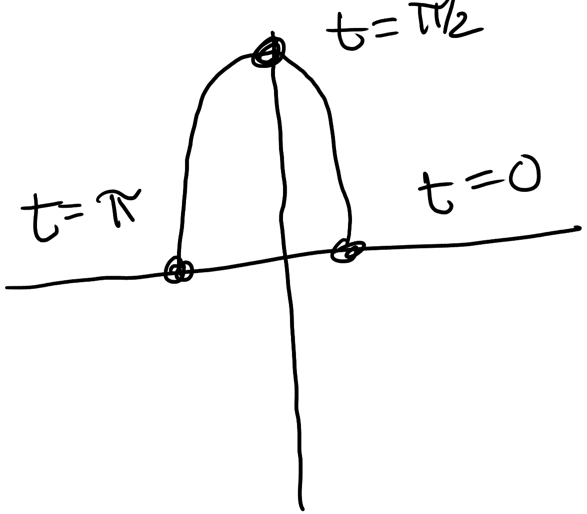
$\frac{1}{5}(y+1) = \sin(t)$

$1 = (\cos(t))^2 + (\sin t)^2 = \left(\frac{1}{2}(x-4)\right)^2 + \left(\frac{1}{5}(y+1)\right)^2$

$= \frac{(x-4)^2}{4} + \frac{(y+1)^2}{25}$

$100 = 25(x-4)^2 + 4(y+1)^2$

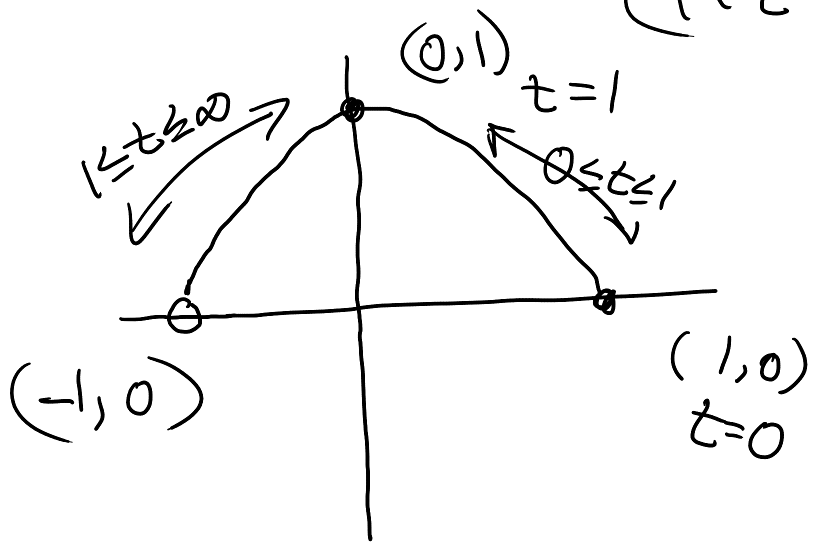
Ellipse



Ex $x(t) = \frac{1-t^2}{1+t^2}$, $y(t) = \frac{2t}{1+t^2}$ $\quad 0 \le t \le \infty$

$$x^2+y^2 = \left(\frac{1-t^2}{1+t^2}\right)^2 + \left(\frac{2t}{1+t^2}\right)^2 = \frac{1-2t^2+t^4}{(1+t^2)^2} + \frac{4t^2}{(1+t^2)^2}$$

$$= \frac{1+2t^2+t^4}{(1+t^2)^2} = \frac{(1+t^2)^2}{(1+t^2)^2} = 1$$



$t=1$ $\quad x(1) = \frac{1-1^2}{1+1^2} = 0$ $\quad y(1) = \frac{2}{1+1^2} = 1$

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} \frac{1-t^2}{1+t^2} = \lim_{t\to\infty} \frac{-t^2}{t^2} = -1$$

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} \frac{2t}{1+t^2} = 0$$

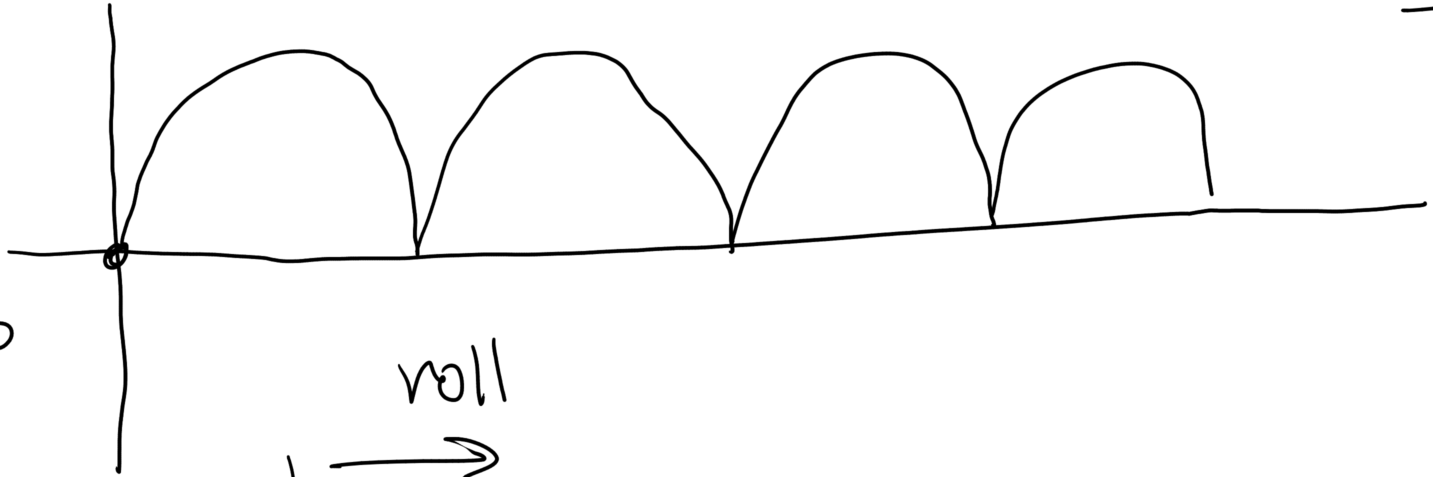
Ex

$$x(t) = t - \sin(t) \qquad 0 \le t \le \infty$$
$$y(t) = 1 - \cos(t)$$

$t=0$ $\quad (x,y) = (x(0), y(0)) = (0, 0)$

cycloid

On YouTube!



$t=0$