Math 290 Thursday January 21 Section 5SLE

Fri - RREF
BYOB
sage
Mon - SLE
Tue - Problems

Solve linear systems

$$\left.\begin{aligned} 2x + 3y + z &= 10 \\ -5x + 8y + 2z &= 3 \end{aligned}\right]$$ System "simultaneous" all true

Language : Proof Technique L

Time flies like an arrow,
fruit flies like a banana.

Equation operations
1) Swap equations
2) Multiply equation by non-zero scalar.
3) Add multiple of an equation to another.

Defn Two systems are **equivalent** if they have the same solution set.

Theorem EOPSS Apply any equation operation, get equivalent systems

Ex

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 3 \\ x_1 + 2x_2 &= 2 \\ 2x_1 + 4x_2 + x_3 &= 6 \end{aligned}$$

$-E_1 + E_2$, $-2E_1 + E_3$ →

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 3 \\ \square \quad -x_2 - x_3 &= -1 \\ -2x_2 - x_3 &= 0 \end{aligned}$$

$-1E_2$ →

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 3 \\ x_2 + x_3 &= 1 \\ -2x_2 - x_3 &= 0 \end{aligned}$$

$2E_2 + E_3$ →

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 3 \\ x_2 + x_3 &= 1 \\ \square \quad x_3 &= 2 \end{aligned}$$

Back substitution

$-E_3 + E_2$, $-E_3 + E_1$ →

$$\begin{aligned} x_1 + 3x_2 \square &= 1 \\ x_2 \square &= -1 \\ x_3 &= 2 \end{aligned}$$

$-3E_2 + E_1$ →

$$\begin{aligned} x_1 \square &= 4 \\ x_2 &= -1 \\ x_3 &= 2 \end{aligned}$$

Only solution $x_1 = 4$, $x_2 = -1$, $x_3 = 2$

Ex

$$\begin{aligned} x_1 + 2x_2 - 8x_3 + 6x_4 &= 10 \\ x_1 + 3x_2 - 11x_3 + 7x_4 &= 12 \\ x_1 \quad\quad - 2x_3 + 4x_4 &= 6 \end{aligned}$$

$-E_1 + E_2$, $-E_1 + E_3$ $\longrightarrow$

$$\begin{aligned} x_1 + 2x_2 - 8x_3 + 6x_4 &= 10 \\ x_2 - 3x_3 + x_4 &= 2 \\ -2x_2 + 6x_3 - 2x_4 &= -4 \end{aligned}$$

$-2E_2 + E_1$, $2E_2 + E_3$ $\longrightarrow$

$$\begin{aligned} x_1 \quad\quad - 2x_3 + 4x_4 &= 6 \\ x_2 - 3x_3 + x_4 &= 2 \\ 0 &= 0 \end{aligned}$$ ← always true

Austin $x_3 = 7$, $x_4 = 3$ $\Rightarrow$ $x_1 = 8$, $x_2 = 20$

Sarah $x_3 = 1$, $x_4 = 2$ $\Rightarrow$ $x_1 = 0$, $x_2 = 3$

Jake $x_3 = 0$, $x_4 = 0$ $\Rightarrow$ $x_1 = 6$, $x_2 = 2$