

Math 290 January 25, 2021

Section TSS

"consistent" a system has at least one solution

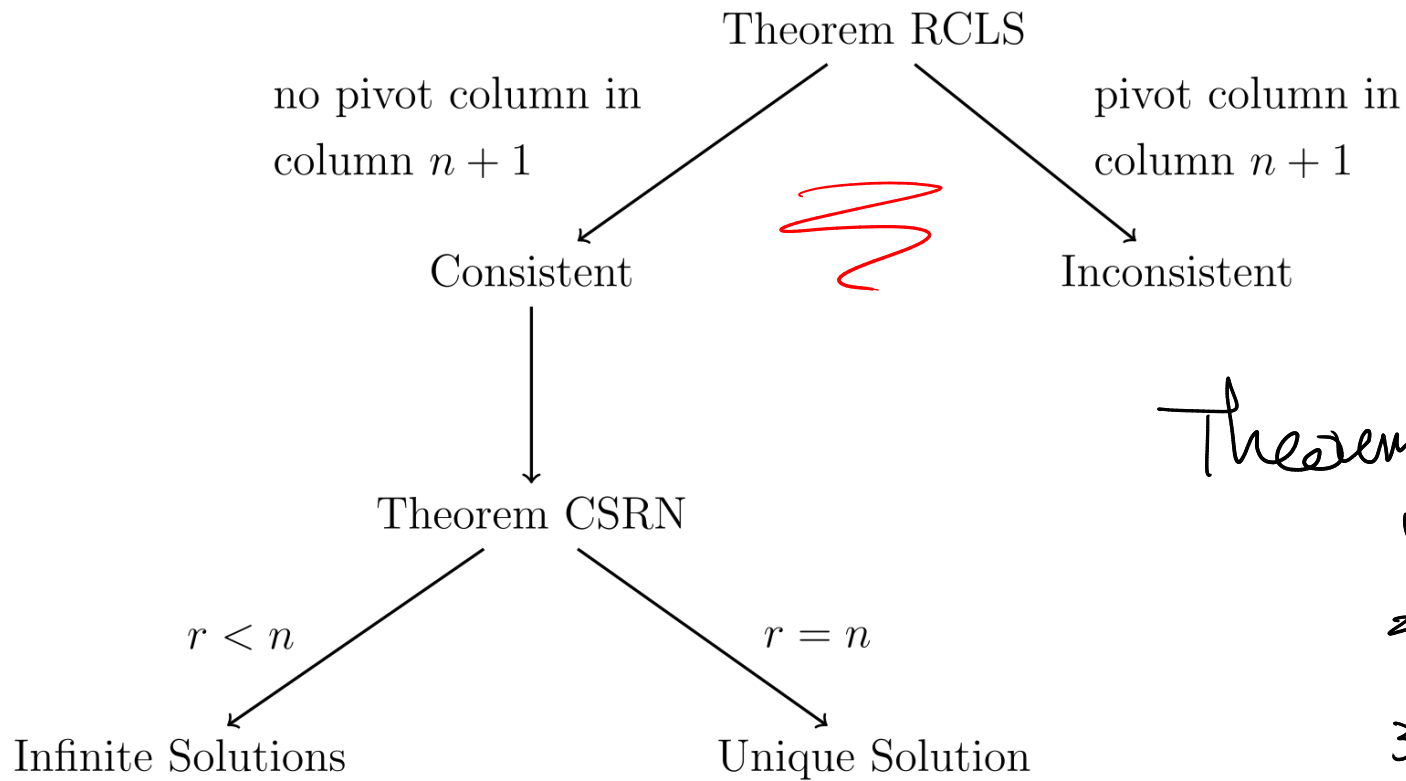
"If column  $i$  is a pivot column then the variable with index  $i$  is a dependent variable"

The Problem Session  
C/NC

Thu HSE

Fri Email PDF  
w/ math

NM



Theorem PSSLS

- 1) no solutions
- 2) one solution
- 3) infinitely many solutions

$r =$  # of leading 1's  
 $=$  # non zero rows  
 $=$  # pivot columns

Ex System of equations w/ augmented matrix

$$\left[ \begin{array}{cccc|c} 1 & 2 & -8 & 6 & 10 \\ 1 & 3 & -11 & 7 & 12 \\ 1 & 0 & -2 & 4 & 6 \end{array} \right] \xrightarrow{\text{RREF}} \left[ \begin{array}{cccc|c} \textcircled{1} & 0 & -2 & 4 & 6 \\ 0 & \textcircled{1} & -3 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Column 5, not pivot

$$r = 2 < 4 = n$$

$\Rightarrow$   
CRN

RCLS

$\Rightarrow$  consistent

infinitely many solutions

Ex System:

$$\left[ \begin{array}{cccc|c} 1 & 0 & 6 & -9 & -4 \\ -2 & 1 & -10 & 23 & 10 \\ 1 & 3 & 12 & 6 & 3 \\ 1 & 1 & 8 & -4 & 1 \end{array} \right]$$

4 equations, 4 variables

$$\xrightarrow{\text{RREF}} \left[ \begin{array}{cccc|c} \textcircled{1} & 0 & 6 & -9 & 0 \\ 0 & \textcircled{1} & 2 & 5 & 0 \\ 0 & 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Column 5 is a pivot column

RCLS  $\Rightarrow$

system inconsistent

Ex

System

$$\left[ \begin{array}{cccccc|c} 1 & 2 & 12 & -1 & 1 & -3 & -5 \\ 0 & 1 & 3 & -1 & -2 & 2 & -13 \\ 1 & 3 & 15 & -1 & 1 & -1 & -11 \\ -2 & -3 & -21 & 3 & 1 & 6 & 14 \\ 1 & 3 & 15 & -1 & 2 & -3 & -8 \end{array} \right]$$

~~RREF~~  
→

$$\left[ \begin{array}{cccccc|c} \textcircled{1} & 0 & 6 & 0 & 0 & -1 & 5 \\ 0 & \textcircled{1} & 3 & 0 & 0 & 2 & -6 \\ 0 & 0 & 0 & \textcircled{1} & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & \textcircled{1} & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

= 5

5  
-6  
1  
3  
0

↑

↑

$r=4 < 6=n$

5 equations, 6 variables

consistent by RCLS ;

free variables  $x_3$  &  $x_6$

$x_1 = 5 - 6x_3 + x_6$

$x_2 = -6 - 3x_3 - 2x_6$

$x_4 = 1 - 4x_6$

$x_5 = 3 + 2x_6$