Math 290 Friday, February 12 Section LDS

Mon – O

Tue – Problems writing V

BREAK

Mon – MO+

RLD on S

Ex $S = \left\{ \begin{bmatrix} 0 \\ -3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -6 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ -1 \\ -6 \end{bmatrix}, \begin{bmatrix} -2 \\ 17 \\ -15 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -7 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 11 \\ -9 \\ 17 \end{bmatrix} \right\} \subseteq \mathbb{C}^4$

$= \{ v_1, v_2, \ldots, v_7 \}$

$\langle S \rangle = ?$

$A = [v_1 | v_2 | \cdots | v_7] \xrightarrow{RREF} \begin{bmatrix} 1 & 2 & 0 & 0 & -3 & 2 & 2 \\ 0 & 0 & 1 & 0 & -4 & -1 & -1 \\ 0 & 0 & 0 & 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Element of $N(A)$

$x_2 = x_5 = x_6 = 0, \; x_7 = 1 \Rightarrow x_1 = -2, \; x_3 = 1, \; x_4 = -3$

$\overset{SLC}{\Longrightarrow}$ $-2v_1 + 1v_3 + -3v_4 + 1v_7 = 0$

$v_7 = 2v_1 - 1v_3 + 3v_4$

Ex $\underbrace{2v_1 + 4v_3 + 5v_6 + 3v_7}_{\in \langle S \rangle} = 2v_1 + 4v_3 + 5v_6 + 3(2v_1 - v_3 + 3v_4) = 8v_1 + v_3 + 9v_4 + 5v_6$

So $\langle S \rangle = \langle \{ v_1, v_2, \ldots, v_6 \} \rangle$ "slimmed down the spanning set"

Element of $N(A)$ $x_2 = x_5 = x_7 = 0$, $x_6 = 1$ $\quad x_1 = -2$, $x_3 = 1$, $x_4 = -1$

$-2\mathbf{v}_1 + \mathbf{v}_3 + -1\mathbf{v}_4 + 1\mathbf{v}_6 = \mathbf{0}$ RLD on S

$\mathbf{v}_6 = 2\mathbf{v}_1 - \mathbf{v}_3 + \mathbf{v}_4$ $\quad \mathbf{v}_6$ "extra"

So $\langle S \rangle = \langle \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\} \rangle$

Again $x_2 = x_6 = x_7 = 0$, $x_5 = 1$ $\quad \mathbf{v}_5 = -3\mathbf{v}_1 - 4\mathbf{v}_3 + 2\mathbf{v}_4$ $\quad \langle S \rangle = \langle \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} \rangle$

Again $x_2 = 1$, $x_5 = x_6 = x_7 = 0$ $\quad \mathbf{v}_2 = 2\mathbf{v}_1$ $\quad \langle S \rangle = \langle \{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4\} \rangle$ ← linearly independent

Theorem BS Put vectors of spanning set in a matrix as columns, RREF, keep vectors that become pivot columns

Bonus: set is linearly independent

Theorem DLDS

$S = \{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_n\}$

S linearly dependent $\iff$ there is $\mathbf{u}_t$ is linear combination of others.

$\Leftarrow$

$$2\begin{bmatrix}1\\10\\100\\1000\end{bmatrix} + 3\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \begin{bmatrix}5\\23\\203\\2003\end{bmatrix} \longrightarrow 2\begin{bmatrix}1\\10\\100\\1000\end{bmatrix} + 3\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + (-1)\begin{bmatrix}5\\23\\203\\2003\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} = \underset{\sim}{0}$$

RLD

Proof ($\Rightarrow$) Assume linear dependence, so there are scalars (← not all zero)

$$a_1 \underline{u}_1 + a_2 \underline{u}_2 + \cdots + a_n \underline{u}_n = \underset{\sim}{0}$$

Say $a_t \neq 0$

$$a_t \underline{u}_t = -a_1 \underline{u}_1 - a_2 \underline{u}_2 - \cdots - a_n \underline{u}_n \quad \text{(no } -a_t u_t\text{)}$$

multiply by $1/a_t$

$$\underline{u}_t = \left(-a_1/a_t\right)\underline{u}_1 + \cdots + \left(\frac{-a_{t-1}}{a_t}\right)\underline{u}_{t-1} + \frac{-a_{t+1}}{a_t}\underline{u}_{t+1} + \cdots + \frac{-a_n}{a_t}\underline{u}_n$$

($\Leftarrow$) one vector is a lin combo of others

$$\underline{u}_t = b_1 \underline{u}_1 + b_2 \underline{u}_2 + \cdots + b_{t-1} \underline{u}_{t-1} + b_{t+1} \underline{u}_{t+1} + \cdots + b_n \underline{u}_n$$

$$\underset{\sim}{0} = b_1 \underline{u}_1 + \cdots + b_{t-1} \underline{u}_{t-1} + (-1)\underline{u}_t + b_{t+1} \underline{u}_{t+1} + \cdots + b_n \underline{u}_n \quad \text{RLD}$$

← nontrivial RLD

So S *is* linearly dependent