Math 290 Thursday, April 1 Section IS

Fri - Problem Session

- BYOB Foreign Country

Mon - SD

Invariant Subspace (of a matrix)

$W \subseteq \mathbb{C}^n$, A $n \times n$

If $\underline{x} \in W$ then $A\underline{x} \in W$

Ex Example TIS

$$A = \begin{bmatrix} -8 & 6 & -15 & 9 \\ -8 & 14 & -10 & 18 \\ 1 & 1 & 3 & 0 \\ 3 & -8 & 2 & -11 \end{bmatrix}$$

$$W = \left\langle \left\{ \begin{bmatrix} -7 \\ -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\} \right\rangle = \langle \{ \underline{w}_1, \underline{w}_2 \} \rangle$$

$$A\underline{w}_1 = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix} = \underline{w}_2$$

$$A\underline{w}_2 = \begin{bmatrix} 5 \\ -2 \\ -3 \\ 2 \end{bmatrix} = (-1)\underline{w}_1 + 2\underline{w}_2$$

$$A(a_1\underline{w}_1 + a_2\underline{w}_2)$$

$$= a_1 A\underline{w}_1 + a_2 A\underline{w}_2$$

$$= a_1(\underline{w}_2) + a_2(-\underline{w}_1 + 2\underline{w}_2) \in W$$

$\mathcal{E}_A(\lambda)$ is an invariant subspace

scalar closure

Grab $\underset{\sim}{x} \in \mathcal{E}_A(\lambda)$ then $A\underset{\sim}{x} = \lambda \underset{\sim}{x} \in \mathcal{E}_A(\lambda)$

scalar

Others: $C(A)$ & $N(A)$ are invariant subspaces

Exercises

$\subsetneq \quad \subsetneq$ "top-out"

Null Spaces of Powers.

$$N(A^k) \subsetneq N(A^{k+1}) \cdots\cdots \subseteq N(A^m) = N(A^{m+1}) \cdots = N(A^n)$$

$\underset{\sim}{x} \in N(A^k)$, know $A^k \underset{\sim}{x} = \underset{\sim}{0}$.

Is $\underset{\sim}{x} \in N(A^{k+1})$? Check $A^{k+1}\underset{\sim}{x} = (AA^k)\underset{\sim}{x} = A(A^k\underset{\sim}{x}) = A\underset{\sim}{0} = \underset{\sim}{0}$

So, yes, $\underset{\sim}{x} \in N(A^{k+1})$

Generalized Eigenspace

$$G_A(\lambda) = \{ \underset{\sim}{x} \mid (A-\lambda I)^k \underset{\sim}{x} = \underset{\sim}{0} \text{ for some } k \}$$

Theorem GENS $\longrightarrow$ $$= N\left((A-\lambda I)^n \right)$$

~~Facts~~ GEIS Generalized eigenspace is invariant

Algebraic multiplicity is dimension of generalized eigenspace.