

Math 290

Thursday, April 15

Section ILT

Poll

Tri-SLT

A)

Mon
IVLT

Tue
Problems
writing

Thu
VR

Fri
MR

Mon
Problems

Tue
Exam LT

B)

Problems

IVLT

Problems
writing

VR

MR

Exam LT

Defn

$T: U \rightarrow V$

T injective (one-to-one)

if whenever

$T(\underline{x}) = T(\underline{y})$, then $\underline{x} = \underline{y}$

Ex Not injective $T: \mathbb{C}^3 \rightarrow \mathbb{C}^4$

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -5x_1 + 4x_2 - 6x_3 \\ -6x_1 + 5x_2 - 7x_3 \\ -x_1 + x_2 - x_3 \\ 3x_1 - 2x_2 + 4x_3 \end{bmatrix}$$

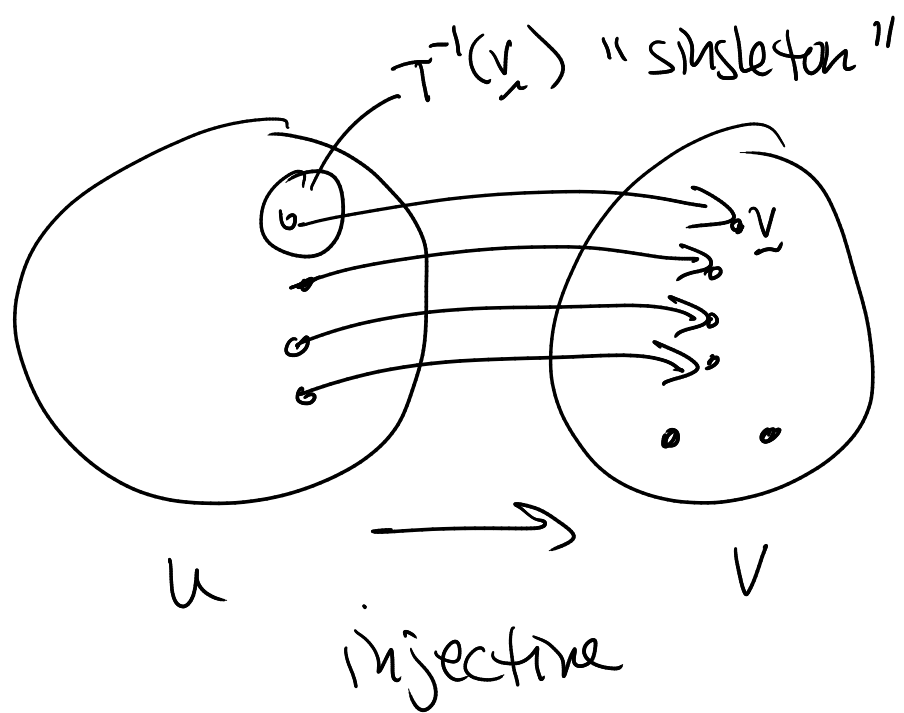
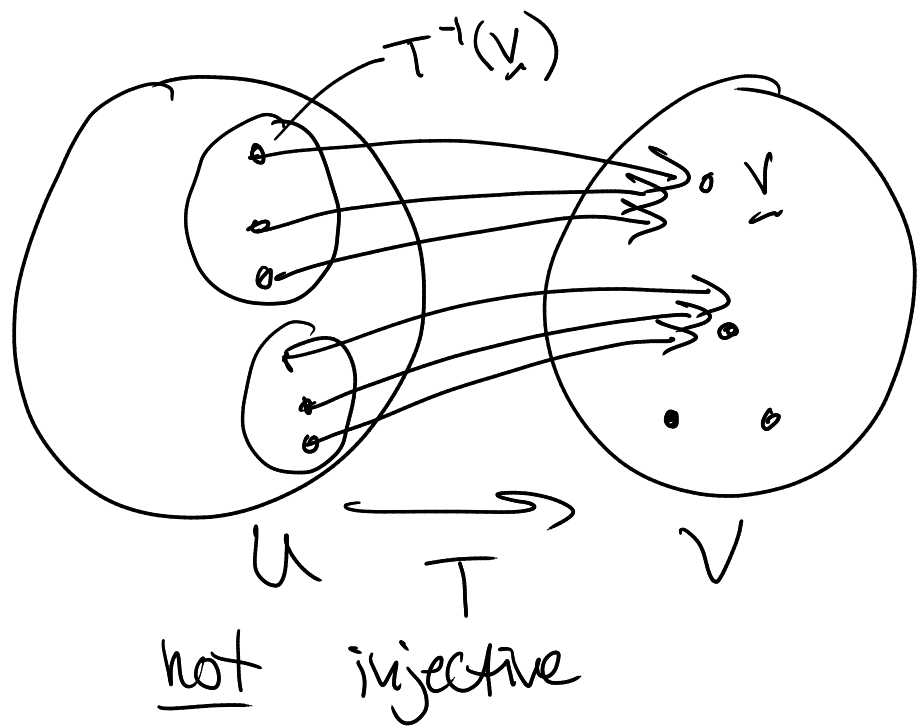
$$T \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{bmatrix} -21 \\ -25 \\ -4 \\ 13 \end{bmatrix}$$

$$K(T) = \left\langle \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} \right\rangle$$

$$T \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \underbrace{\quad}_{\tilde{z}}$$

$$T \begin{pmatrix} -3 \\ -3 \\ 4 \end{pmatrix} = \begin{bmatrix} -21 \\ -25 \\ -4 \\ 13 \end{bmatrix}$$

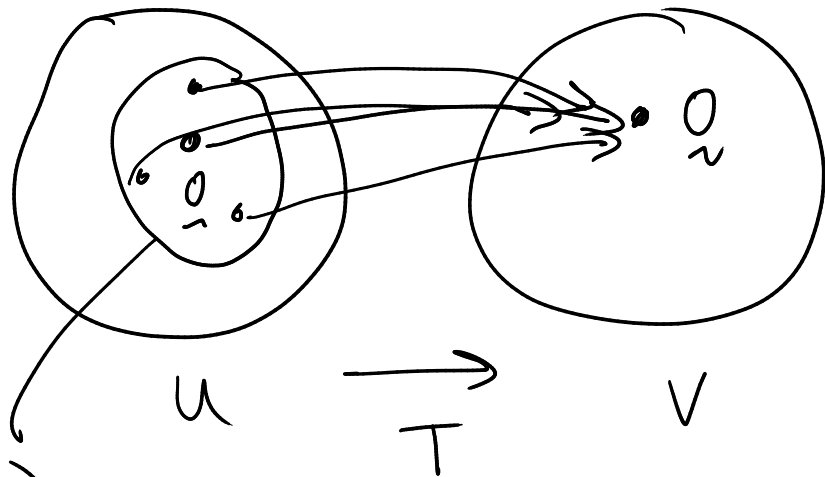
$$\begin{aligned} T \left(\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} -3 \\ -3 \\ 4 \end{bmatrix} \right) &= T \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 2 T \begin{pmatrix} -3 \\ -3 \\ 4 \end{pmatrix} \\ &= T \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 2 \cdot 0 \\ &= T \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \end{aligned}$$



Kernel
 $T: U \rightarrow V$

$$K(T) = \{ \underline{u} \in U \mid T(\underline{u}) = \underline{0} \}$$

- ① Subspace, of U .
- ② $K(T) = T^{-1}(\underline{0}_V)$
- ③ T injective $\iff K(T) = \{ \underline{0} \}$ (Theorem KILT)



$K(T)$
 $T^{-1}(0_V)$

\cong

$S: P_2 \rightarrow M_{22}$

$$S(a+bx+cx^2) = \begin{bmatrix} a-2b+c & -a+3b \\ 2b+3c & a-5b-4c \end{bmatrix}$$

$K(S)?$

$$S(a+bx+cx^2) = \underline{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \\ \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{aligned} a-2b+c &= 0 \\ -a+3b &= 0 \\ 2b+3c &= 0 \\ a-5b-4c &= 0 \end{aligned}$$

$$\Rightarrow a=b=c=0 \Rightarrow K(S) = \{0+0x+0x^2\}$$

$\Rightarrow S$ is injective
 KILT