Math 290 Friday, April 30 Section CB

RQ - Scores posted
spreadsheet (%)

Mon - Problem Session
Writing R
0/1/2

Tue - Exam R
- no proofs

Ex $T : M_{22} \to P_2$

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (2a+b+3c-2d) + (5a+3b+7c-4d)x + (a+b+c)x^2$$

$$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \qquad D = \{1, x, x^2\}$$

$$C = \left\{ \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix} \right\} \qquad E = \{1, 1+x, 1+x+x^2\}$$

Change - of - Basis

V - vector space (one only)

B, C - two bases

$I_V : V \to V \qquad I_V(v) = v$

$M^{I_V}_{B,C} = C_{B,C}$

Theorem CB

$\rho_C(\underset{\sim}{v}) = C_{B,C}\, \rho_B(\underset{\sim}{v})$

Fact $\quad C_{C,B} = C_{B,C}^{-1}$

Theorem MRCB

$T: U \to V$

bases $\quad B, C \quad D, E$

1 $\quad$ 2 $\quad$ 3

$$M^T_{B,D} = C_{E,D}\, M^T_{C,E}\, C_{B,C}\left(\rho_B(\underset{\sim}{u})\right)$$

$$D \xleftarrow{T} B \;=\; \underbrace{D \leftarrow E \xleftarrow{T} C \leftarrow B}\leftarrow \underset{\sim}{u}$$

Specialize to $T: V \to V$
$\quad B, C$

Substitute
$V \to U$
$D \to B$
$E \to C$

Theorem SCB $\quad M^T_{B,B} = C_{C,B}\, M^T_{C,C}\, C_{B,C}$

$$= C^{-1}_{B,C}\, M^T_{C,C}\, C_{B,C}$$

similarity

Eigenvalues of a Linear Transformation

$$T : V \to V \qquad T(\underset{\sim}{v}) = \underbrace{\lambda \underset{\sim}{v}}_{\in \text{ codomain}}$$

$\underset{\sim}{v} \in$ domain

☆ Build any matrix representation of T, the eigenvalues of that matrix are the eigenvalues of T. ☆

☆ A matrix representation relative to a basis of eigenvectors will be diagonal. ☆

Ex

$$M^T_{B,D} = \begin{bmatrix} 2 & 1 & 3 & -2 \\ 5 & 3 & 7 & -4 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$ "on sight"

$$M^T_{C,E} = \begin{bmatrix} -5 & -12 & -5 & -12 \\ 10 & 20 & 12 & 16 \\ 0 & 2 & -1 & 4 \end{bmatrix}$$ "with care"

$$C_{C,B} = \begin{bmatrix} 1 & 2 & 1 & 2 \\ -2 & -3 & -3 & -2 \\ 1 & 3 & 1 & 4 \\ -1 & 0 & -2 & 3 \end{bmatrix}$$ easier

$$C_{B,C} = C_{C,B}^{-1}$$

$$C_{E,D} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_{D,E} = C_{E,D}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M^T_{B,D} = C_{E,D}\, M^T_{C,E}\, C_{B,C}$$

$3 \times 4 \qquad 3 \times 3 \qquad 3 \times 4 \qquad 4 \times 4$

$\underline{D,B} \qquad D,E \quad E,C \quad C,B$