Math 390 Friday January 22 Linear Transformations

$T: U \to V$

1) $T(u_1 + u_2) = T(u_1) + T(u_2)$

2) $T(\alpha u) = \alpha T(u)$

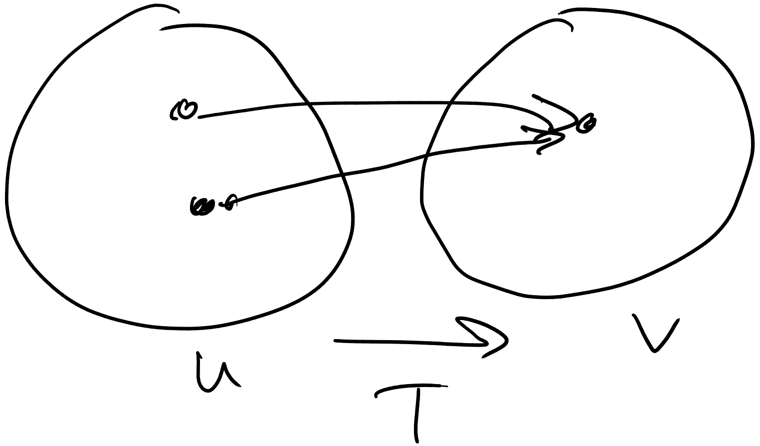
$T(\cdot)$

Structure-preserving

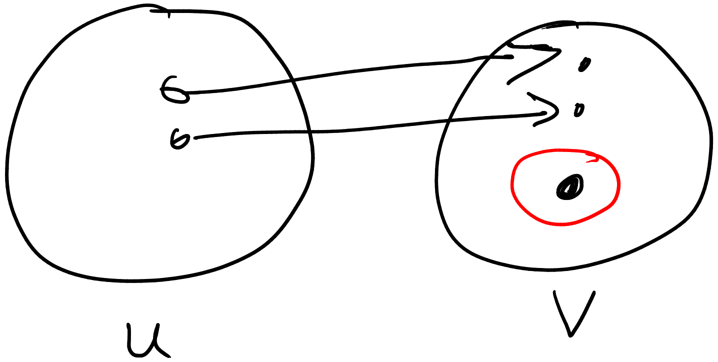
"morphisms"

Injective (1-1) $T(u_1) = T(u_2) \Rightarrow u_1 = u_2$

Surjective (onto) Grab $v \in V \Rightarrow$ there exists $u \in U$, $T(u) = v$



not injective



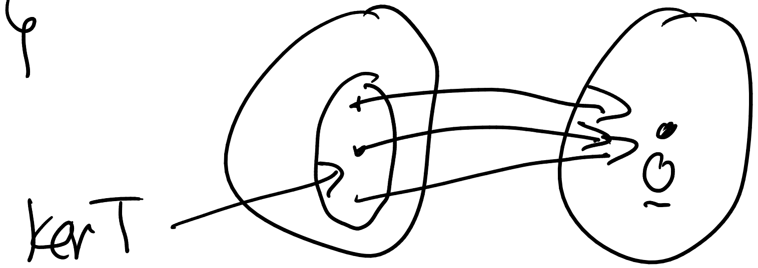
not surjective

Ex $T : P_3 \to M_{22}$

$$T(a+bx+cx^2+dx^3) = \begin{bmatrix} a-b+3d & -a+2b-c-6d \\ a+b-c+2d & -b+c+4d \end{bmatrix}$$

Kernel $\quad K(T) = \{ \underset{\sim}{x} \mid T(\underset{\sim}{x}) = \underset{\sim}{0} \}$

subspace



$$\begin{bmatrix} & & \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\ker(T) = \{ \underset{\sim}{0} \}$

$\Rightarrow T$ injective

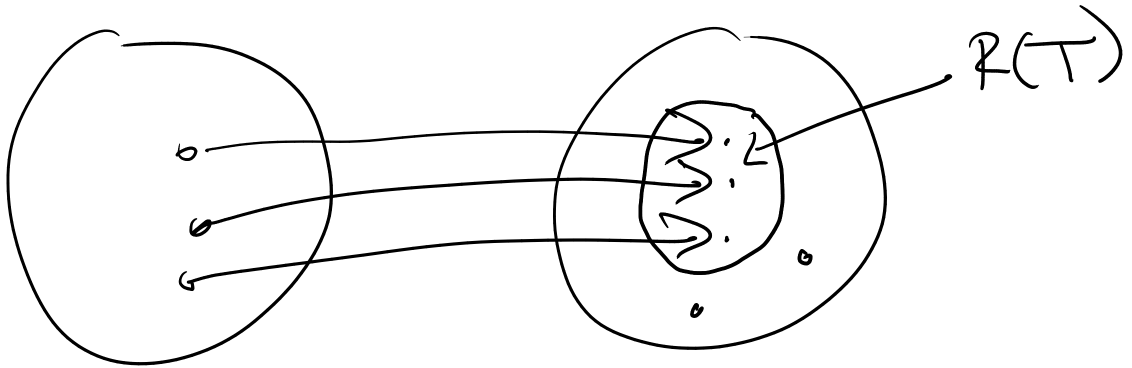
Homogeneous system, 4 equations

4 variables $\Rightarrow$ solution

$a=0$
$b=0$
$c=0$
$d=0$

Range $R(T) = \{ T(\underset{\sim}{u}) \mid \underset{\sim}{u} \in U \}$ subspace of V

<u>Theorem</u> T surjective $\Longleftrightarrow$ $R(T) = V$



$$\underbrace{r(T)}_{\dim(R(T))} + \underbrace{n(T)}_{\dim(K(T))} = \dim(U)$$

$$r(T) + 0 = \dim(P_3) = 4$$

$$\Rightarrow r(T) = 4, \quad R(T) \subseteq \underbrace{M_{22}}_{\dim 4}$$

EDYES

$$\Rightarrow R(T) = M_{22}$$

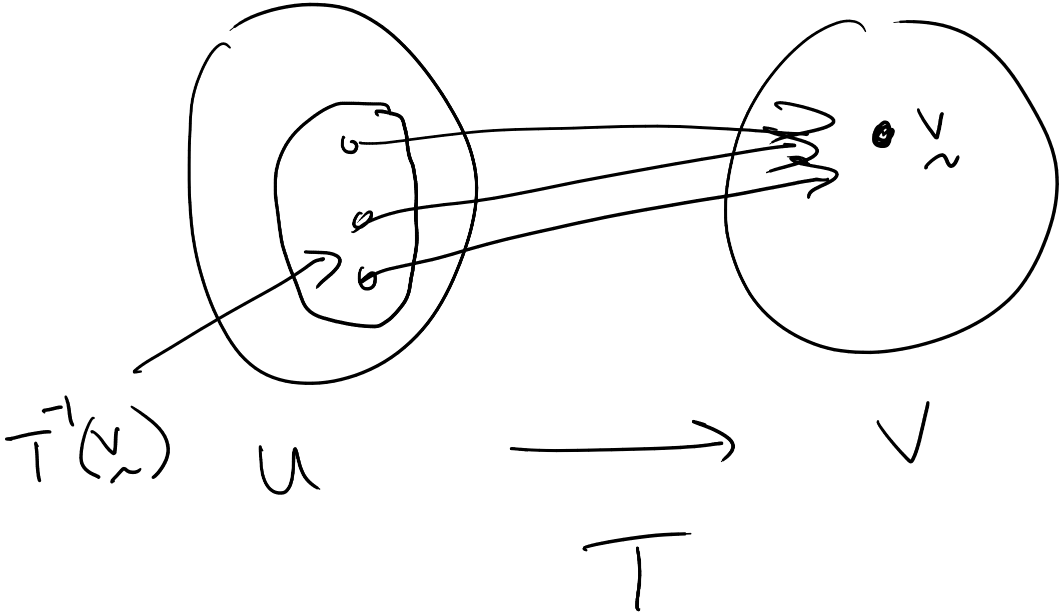
$$\Rightarrow T \text{ surjective}$$

T injective & surjective

$\Rightarrow T$ bijective

Preimages Grab $\underset{\sim}{v} \in V$ define $T^{-1}(\underset{\sim}{v}) = \{\underset{\sim}{u} \in U \mid T(\underset{\sim}{u}) = \underset{\sim}{v}\}$

$\ker(T) = T^{-1}(\underset{\sim}{0})$



Theorem KPI Suppose $T(\underset{\sim}{u}) = \underset{\sim}{v}$

then $T^{-1}(\underset{\sim}{v}) = \underset{\sim}{u} + K(T)$

T surjective $\Rightarrow T^{-1}(\ )$ non empty

T injective $\Rightarrow T^{-1}(\ )$ size at most 1

Bijective $\Rightarrow T^{-1}(\ )$ singleton

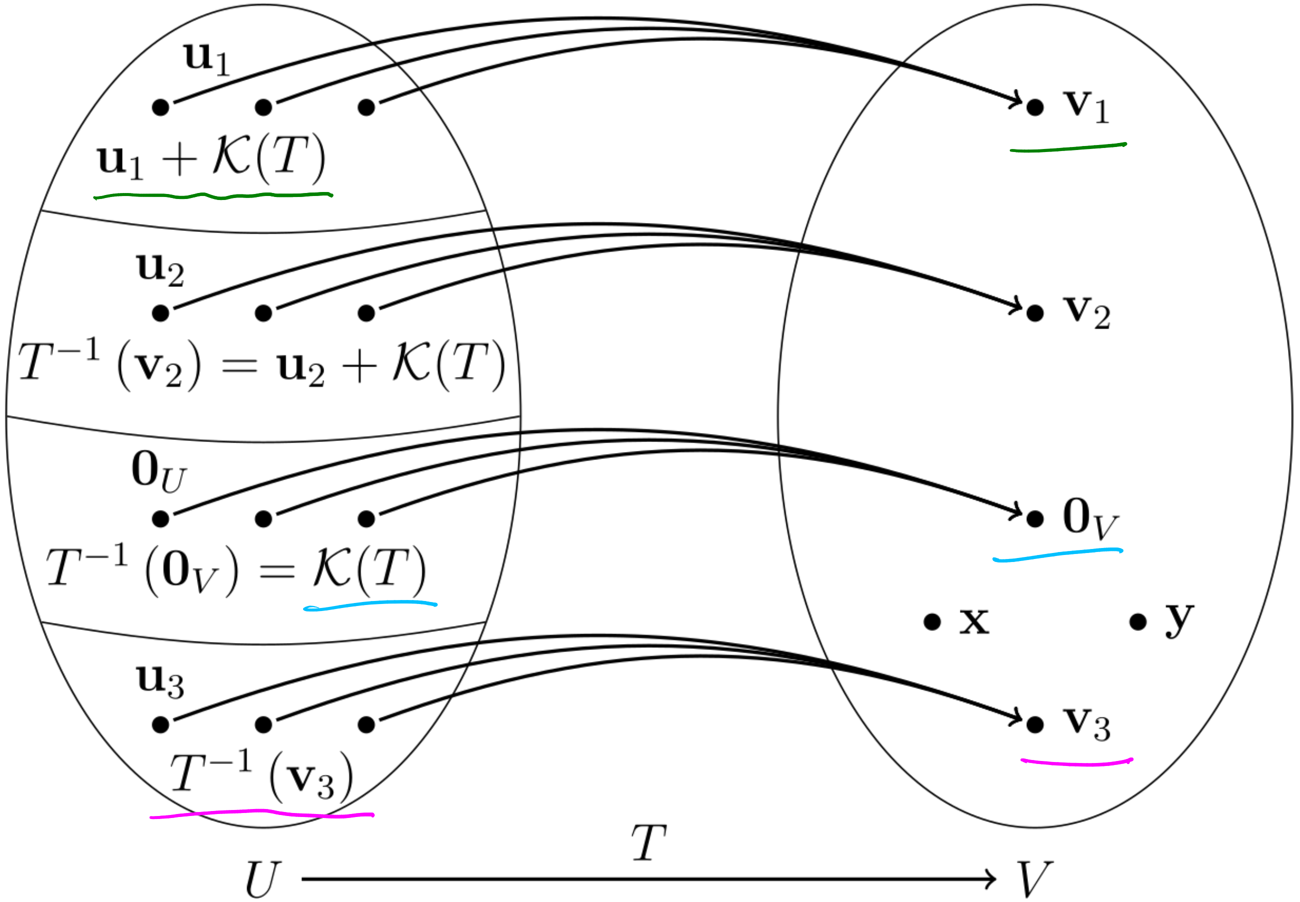
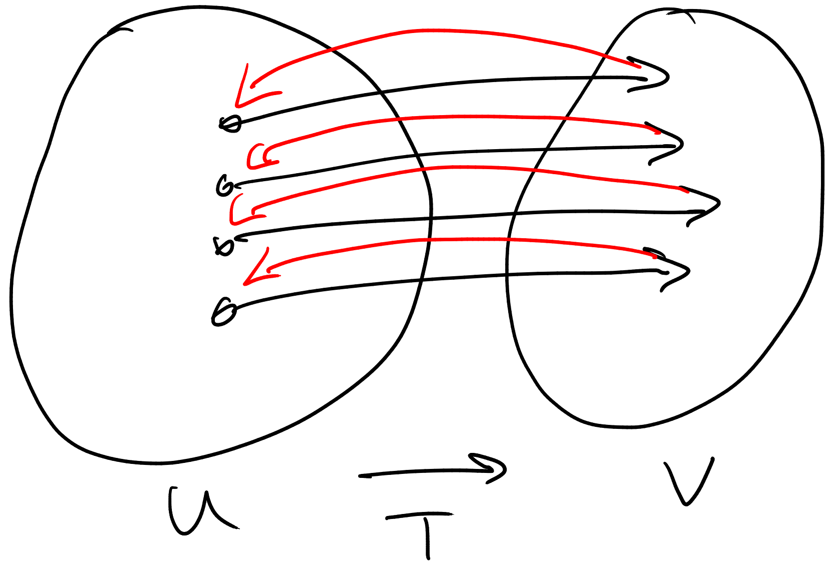


Figure KPI

Preimages : Equivalence Relation



Bijection

Inverse (Linear Transformation)

$T^{-1}\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right) = \{-6-4x-8x^2+x^3\}$ Preimages

$T^{-1}\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right) = \{-6-3x-7x^2+x^3\}$

$T^{-1}\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right) = \{1+x+x^2\}$

$T^{-1}\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right) = \{-5-2x-5x^2+x^3\}$

$T^{-1}\left(\begin{bmatrix}3&2\\-6&4\end{bmatrix}\right) = T^{-1}\left(3\begin{bmatrix}1&0\\0&0\end{bmatrix}+2\begin{bmatrix}0&1\\0&0\end{bmatrix}+-6\begin{bmatrix}0&0\\1&0\end{bmatrix}+4\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)$

$= 3(\quad) + 2(\quad) + -6(\quad) + 4(\quad)$

inverse l.t.