Math 390 Thursday, January 28 FCLA EE (revised)

Eigenvalues & Eigenvectors : A square matrix

$A\underset{\sim}{x} = \lambda \underset{\sim}{x}$ $\quad \lambda \in F \leftarrow$ algebraically closed ?

Polynomials of a Matrix

Ex $A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ $\quad A^2 = \begin{bmatrix} -2 & 7 \\ 1 & -1 \end{bmatrix}$ $\quad A^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$p(x) = 3x^2 - x + 8$ $\quad p(A) = 3\begin{bmatrix} -2 & 7 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} + 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 18 \\ 4 & 3 \end{bmatrix}$

Theorem PMC

$p(A)\,q(A) = q(A)\,p(A)$

Theorem EMHE $\quad A \quad n \times n$

Grab $\underset{\sim}{x} \neq \underset{\sim}{0}$. (anything!) $\quad \underset{\sim}{x} = \underset{\sim}{e}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

$A^0 \underset{\sim}{x},\ A^1 \underset{\sim}{x},\ A^2 \underset{\sim}{x},\ \dots,\ A^n \underset{\sim}{x}$ $\quad$ n+1 vectors from F^n

theorem MVSLD $\Rightarrow$ set linearly dependent

"shortest" RLD

$$a_0 A^0 \underset{\sim}{x} + \cdots + a_{m-1} A^{m-1} \underset{\sim}{x} + \underline{1} \quad A^m \underset{\sim}{x} = \underset{\sim}{0}$$

$$p(x) = x^m + a_{m-1} x^{m-1} + \cdots + a_1 x + a_0$$

$$= (x - b_1)(x - b_2) \cdots (x - b_m) \qquad (F = \mathbb{C}?)$$

$$p(A)\underset{\sim}{x} = \underset{\sim}{0}$$

$$\underbrace{(A - b_1)\underbrace{(A - b_2) \cdots (A - b_m)\underset{\sim}{x}}_{\neq \underset{\sim}{0}}}_{\underset{\sim}{0}} = \underset{\sim}{0}$$

$$\underset{\sim}{z} = (A - b_3) \cdots (A - b_m)\underset{\sim}{x} \neq \underset{\sim}{0}$$

$$(A - b_2)\underset{\sim}{z} = \underset{\sim}{0}$$

$$A\underset{\sim}{z} - b_2 \underset{\sim}{z} = \underset{\sim}{0}$$

$$A\underset{\sim}{z} = b_2 \underset{\sim}{z}$$

- b_2 eigenvalue
- $\underset{\sim}{z}$ eigenvector

Subsection CEE Eigenspaces

Subsection ECEE

Ex Companion matrix $p(x) = x^5 - 6x^3 - 27x - 3$

nobody knows the roots

$$\begin{bmatrix} 0 & & 0 & 3 \\ 1 & 0 & & 27 \\ & 1 & \ddots & -0 \\ & 0 & 1 \quad 0 & 6 \\ & & 1 & -0 \end{bmatrix} \longleftarrow \text{eigenvalues are roots of}$$

Theorem ECM

TRIANGULAR MATRICES

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 62 & 3 \\ 0 & 0 & 304 \end{bmatrix}$$